

Optical BEAMTAP Beamforming and Jammer Nulling System for Phased Array Antennas

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Abstract - We present a new approach to receive mode broadband beamforming and jammer nulling for large adaptive phased arrays as well as its efficient and compact optical implementation. This Broadband Efficient Adaptive Method for True-time-delay Array Processing (BEAMTAP) algorithm decreases the number of tapped-delay-lines required to process an N -element phased array antenna from N to only 2, producing an enormous savings in delay-line hardware (especially for large broadband arrays), while still providing the full NM degrees of freedom of a conventional N element time-delay-and-sum beamformer that requires N tapped-delay-lines with M taps each. This allows the system to fully and optimally adapt to an arbitrarily complex spatio-temporal signal environment that can contain broadband signals of interest, as well as interference sources and narrowband and broadband jammers—all of which can arrive from arbitrary angles onto an arbitrarily shaped array—thus enabling a variety of applications in radar, sonar, and communication. This algorithm is an excellent match with the capabilities of RF photonic systems using gratings in a photorefractive crystal as adaptive weights, a traveling wave detector for generating time-delay, and an acousto-optic device to control weight adaptation. Because the number of available adaptive coefficients in a photorefractive crystal is as large as 10^9 , these photonic systems can adaptively control arbitrarily large 1-D or 2-D phased arrays which are well beyond the capabilities of conventional RF and real-time digital signal processing techniques, or alternative photonic techniques.

Keywords: Adaptive phased array, true-time-delay, optical processing, photorefractive crystals, acousto-optics.

1 Introduction

Conventional broadband time-domain beamformers for phased array processing of radar signals require one tapped-delay-line (TDL) for each antenna element to avoid beam squint, which is the undesired rotation of the angular receptivity pattern of the antenna with frequency. Often these TDLs are implemented with digital delay lines for narrowband systems (up to a few MHz bandwidth). To process signals with up to about a 1 GHz bandwidth, TDLs can be implemented with ultrasonic delay-lines, which can be conveniently tapped by acousto-optic diffraction.¹ At very high frequencies (perhaps even up to 100 GHz) fiber optic TDLs have been proposed.²⁻⁴ Large arrays can have up to 1000 elements for 1-dimensional arrays and up to 100×100 elements for 2-dimensional arrays, requiring $N = 10^3 - 10^4$ broadband TDLs, which is both expensive and technologically challenging. These TDLs are necessary to avoid beam squint, which is especially troublesome in large arrays for large fractional bandwidth applications. In order to obtain squint-free octave-bandwidth beamforming over a half-space for a half-wavelength spaced array, each TDL needs at the very minimum as many complex taps as there are elements in a 1-D phased array or elements along the longest diagonal for a 2-D array. More taps are required, however, for the equalization of antenna frequency response ripples, compensation for mutual coupling, or multipath processing.

To avoid the necessity for a large number of TDLs, many phased arrays are operated in a narrowband mode in which each antenna output is simply multiplied by a single complex coefficient for beamforming, as shown in Figure 1a. However, when the fractional bandwidth $F = B/f_0$ (where B is the bandwidth and f_0 is the center frequency), exceeds the spatial array resolvability $Q = \lambda_m / L \sin \theta_{max}$ (where L is the maximum array aperture which equals $L = Nd$ for an N -element 1-D array of element spacing d , $\lambda_m = c/f_{max}$, and θ_{max} is the maximum angle from boresight over which the array must steer), then a plane wave pulse arriving at a large angle θ from boresight takes a time $T = \frac{L}{c} \sin \theta$ to propagate across the array aperture, and this delay is resolvable within the bandwidth of the system. In this case, when $F > Q$, the antenna array function is significantly altered as the frequency changes across the bandwidth B , resulting in an undesired angular rotation of the main beam and an additional undesired rotation of the nulls of the antenna pattern.

In current implementations of antenna beamformers, when broadband operation is required (the condition where $F > Q$), a weighted TDL with at least $M = \sin \theta_{max} BL/c$ taps is required at every antenna element, as shown in Figure 1b. As an example, for a 1-D array with $N = 1000$, a center frequency of 2 GHz, a processing bandwidth of 1 GHz ($F = 1/2$), a desired angular coverage from boresight to both endfire directions ($\theta_{max} = 90^\circ$), with elements which are spaced at $\lambda_m/2$ (where λ_m is the minimum detectable wavelength of the array), $M = 200$ complex taps are required per array element. This results in a requirement of $NM = 200,000$ complex taps for the entire 1-D array. For a 100×100 2-D array operating at the same frequencies, 56 complex taps are required per antenna element, and a total of 560,000 complex taps are needed for beamforming without squint. In the alternative frequency-domain implementation, a temporal Fourier transform of each antenna output is implemented digitally with an array of N fast Fourier transform (FFT) modules and used to produce $M = BL/c$ narrow band systems, each of which is steered with a straight forward complex multiplier operating at every antenna element at each resolvable

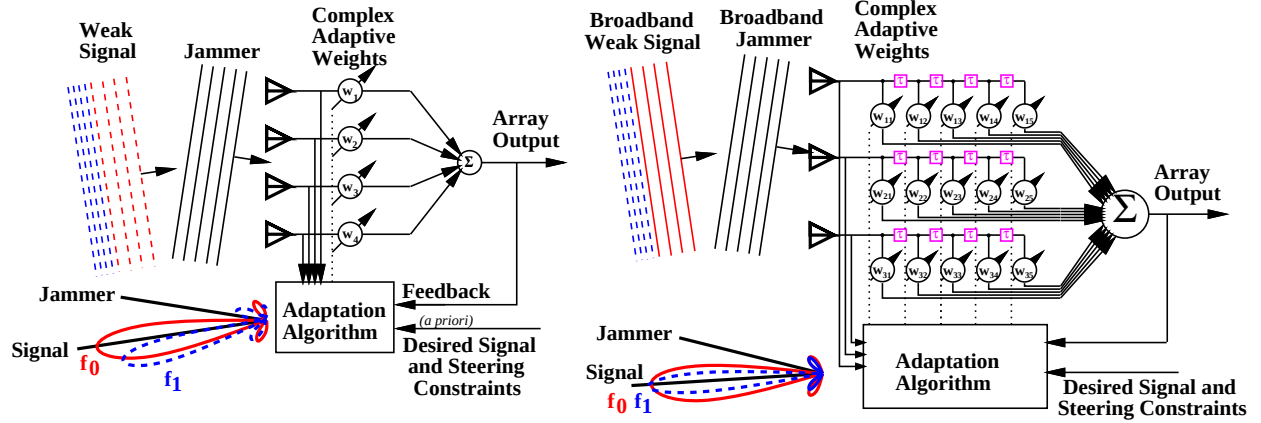


Figure 1: a) Narrowband phased array that suffers from frequency-dependent beam squint. b) Conventional broadband time-delay-and-sum beamforming algorithm illustrating the requirement for one tapped-delay-line at each antenna element in order to eliminate beam squint of the main beam and the nulls.

frequency. After narrowband beamforming in each frequency bin, the desired broadband signal is reconstructed by an inverse FFT of the summed narrowband outputs. However, the frequency domain implementation ignores correlations between frequency components (i.e., it assumes an on-diagonal block Toeplitz correlation structure) that can produce erroneous results in certain types of broadband scenarios. In addition, this frequency domain system has as many FFT modules as conventional time-domain beamformers have delay lines (N), and the FFT modules are far more complex to implement than a delay line, especially for real-time operation.

In this paper we describe a new, more efficient algorithm for broadband time domain beamforming that requires only 1 input tapped-delay-line detector, and one output tapped-delay-line modulator for weight programming of arbitrarily large arrays. This BEAMTAP system makes broadband beamforming viable for large arrays and is well matched to implement with RF photonic hardware. In addition, BEAMTAP is compatible with the real-time calculation of the required $MN = 2F/Q^2$ adaptive weights that encompass the necessary degrees of freedom to nearly optimally beamsteer and null rotate without squint in an arbitrarily complex spatio-temporal signal environment.

We begin by comparing the conventional time-domain beamformer with the new hardware-efficient BEAMTAP architecture, and we show their mathematical equivalence. Then the proposed RF photonic implementation utilizing a fiber-remoted coherent phased array, photorefractive crystal, traveling wave detector, and acousto-optic device is outlined. The details of the RF offset frequencies, grating recording, coherent interferometric detection, and closed loop optical system operation are then derived in Sec. 3. Simulations are presented that demonstrate the operations of TTD main-beam forming and jammer nulling for farfield broadband signals. Finally, we conclude with a discussion on the benefits and challenges of the proposed BEAMTAP optical array processing system when compared to other optical approaches and digital signal processing implementations.

2 Optical Phased Array Radar Processing

There have been numerous approaches to optically implementing adaptive phased-array phase-only beamsteering, all of which unfortunately suffer from beam squint for broadband signals.^{5,6} Previous optical implementations of broadband adaptive beamforming^{7–10} avoided beam squint by using multi-channel acousto-optic delay-lines to implement a TDL for every antenna element input, but are limited to 32 channels because of the limits of AO technology.¹¹ Fiber optic TTD beamforming networks have been employed to bring the main beam onto the apparent array boresight where there is no frequency dependent beam squint,^{2,3,12,13} though squint is still present in the sidelobes and nulls. Although they have the potential to be scanned rapidly, these systems are not adaptive and are only able to process linear or planar arrays and not conformal, irregular, or dynamically flexing arrays. In addition, these fiber-based system do not provide for a mechanism of adaptively weighting the array function to null out jammers or simultaneously point multiple beams. Moreover, fiber optic TTD networks, even the elegant wavelength-tuned dispersive,¹⁴ fiber prism,³ or grating-reflective approaches,¹⁵ can create extreme fiber management problems.

We have previously developed adaptive phased array processing systems using photorefractive crystals as the adaptive weights that achieved 45 dB of narrowband jammer nulling, and had the potential capability for TTD processing, but in a complex frequency domain implementation.¹⁶ A further drawback of our previous system was that to directly perform true-time-delay beamforming on large arrays in the optical domain it was necessary to incorporate as large an optical tapped-delay-line with as large an optical path length as the size of the phased array—yet over this propagation distance the short optical wavelength makes diffractive spreading a significant problem unless re-imaging (as in a stable Herriot cell resonator) or fiber waveguiding is utilized. Our previous system utilized wedged or tilted fabry perot etalons to incorporate this delay (which was manifested as arrays of frequency selective bandpass filters with a resolution inverse to the required delay); however, achieving such resolution on parallel arrays of signals presented a significant engineering challenge. The photonic implementation of the BEAMTAP network presented here is a substantial simplification and outgrowth of this previous work and represents an attractive alternative to the limitations of previous optical approaches.¹⁷ The key is the use of a single traveling wave detector such as a 1-dimensional time-delay-and-integrate charge-coupled device (TDI CCD)^{18,19} or a photoconductive traveling fringes detector^{20,21} to allow the generation of the necessary tap-in time-delays in the electrical (rather than the optical) domain, while the tap-out delay line can be implemented compactly using acoustic delay lines such as acousto-optic Bragg cells for the input. However, all of the adaptive timing, phasing, and weighting is still done in the optical domain.

2.1 Conventional Time-Delay-and-Sum Adaptive Beamformer

In conventional time domain beamforming as shown schematically in Fig. 1 and with the details of the LMS algorithm in Fig. 2, each of the N antenna outputs $s_n(t)$ (for $n = 1, \dots, N$) is input to a TDL, which is represented for discrete delays as $s_n(t - m\tau)$ (for $m = 0, \dots, M - 1$), where the temporal spacing between taps is τ and the total delay of the TDL is $T \equiv$

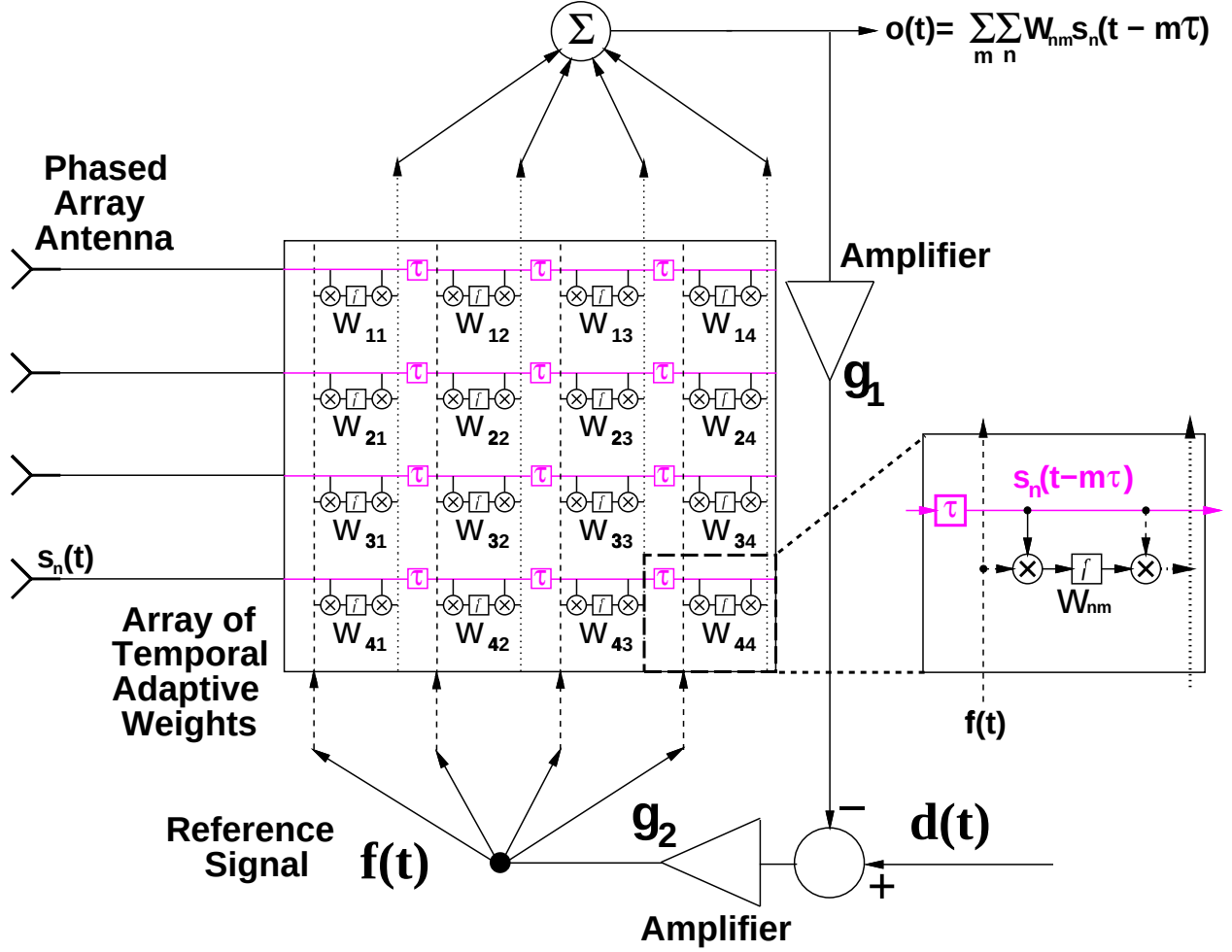


Figure 2: Conventional time-delay-and-sum approach to adaptive array processing for broadband squint-free true-time-delay beamforming using least-mean-square (LMS) adaptation based on a desired signal and a simple correlation-cancellation-loop feedback, where the desired signal d_t is subtracted from the output signal o_t to produce the feedback signal f_t .

$(M-1)\tau = M'\tau$ where M' is the total number of time delays, and represented in continuous notation as $s_n(t - t')$ for $0 < t' < T$. The output consists of a sum of each antenna output causally convolved with finite impulse responses W_{nm}^* of duration T

$$o(t) = \sum_{n=1}^N \sum_{m=0}^{M-1} s_n(t - m\tau) W_{nm}^*. \quad (1)$$

The weighted impulse response filter W_{nm}^* of each antenna element is usually calculated by an adaptive algorithm, and for the common least mean square (LMS) style of algorithms^{22, 23} the weights are calculated as a correlation coefficient between each delayed antenna signal and a signal $f(t)$ —which is either derived from a feedback loop or is an *a priori* known reference signal. The most common form of error driven feedback is when $f(t) = d(t) - o(t)$

for some desired signal $d(t)$. Thus the weighted impulse response filter is given by

$$W_{nm}^* = \int_{-\infty}^t s_n^*(t_1 - m\tau) f(t_1) dt_1. \quad (2)$$

The time dependence of the weights due to adaptation up through time t is very slow and can often be assumed constant over the duration of the convolution with the signals. Given this form of the adaptive weight matrix, the output can be written as

$$\begin{aligned} o(t) &= \sum_{n=1}^N \sum_{m=0}^{M-1} s_n(t - m\tau) W_{nm}^* \\ &= \sum_{n=1}^N \sum_{m=0}^{M-1} s_n(t - m\tau) \int_{-\infty}^t s_n^*(t_1 - m\tau) f(t_1) dt_1 \\ &= \sum_n \sum_m s_n(t - m\tau) \int_{-\infty}^t s_n^*(t_1 - m\tau) [d(t_1) - o(t_1)] dt_1. \end{aligned} \quad (3)$$

Solving for the output $o(t)$ can be accomplished using a frequency domain approach which yields a power nulling spatio-temporal filter. We will show below a nearly identical output for our new BEAMTAP beamformer that requires only 2 TDLs in comparison with the N TDLs required by this LMS time-delay-and-sum conventional approach.

2.2 BEAMTAP Algorithm

In the new BEAMTAP algorithm shown in Fig. 3, each antenna element output $s_n(t)$ is multiplied at every resolvable instant of time t by a linear array of weights W_{nm}^* , which are located along the corresponding row of the weight matrix. Each product is summed along the columns with the corresponding products from the other rows produced by the other array elements, and the resulting sum is input to a tap-in scrolling delay-line, whose inputs at the m th tap-in position are given by a weighted sum over undelayed signals

$$Y_m(t) = \sum_{n=1}^N s_n(t) W_{nm}^*. \quad (4)$$

As the tap-in delay line spatially scrolls in time by increments of τ , it accumulates the appropriate contributions from each column sum position, and after a particular moving sum traverses the entire delay line aperture, the resulting output is the desired multiply-weighted and delayed sum from the array

$$o(t) = \sum_{m=0}^{M-1} Y_m(t - m\tau) = \sum_{m=0}^{M-1} \sum_{n=1}^N s_n(t - m\tau) W_{nm}^*. \quad (5)$$

Thus from any antenna element there is a path into the delay line at a position such that the output has the appropriate delay, and this is exactly equivalent to the conventional time domain beamformer given in Eq. 1. In a sense, we have simply commuted the linear operation of multiple TDLs which have one weighted output from each tap position into a single multiple input delay line which addresses the same array of weights.

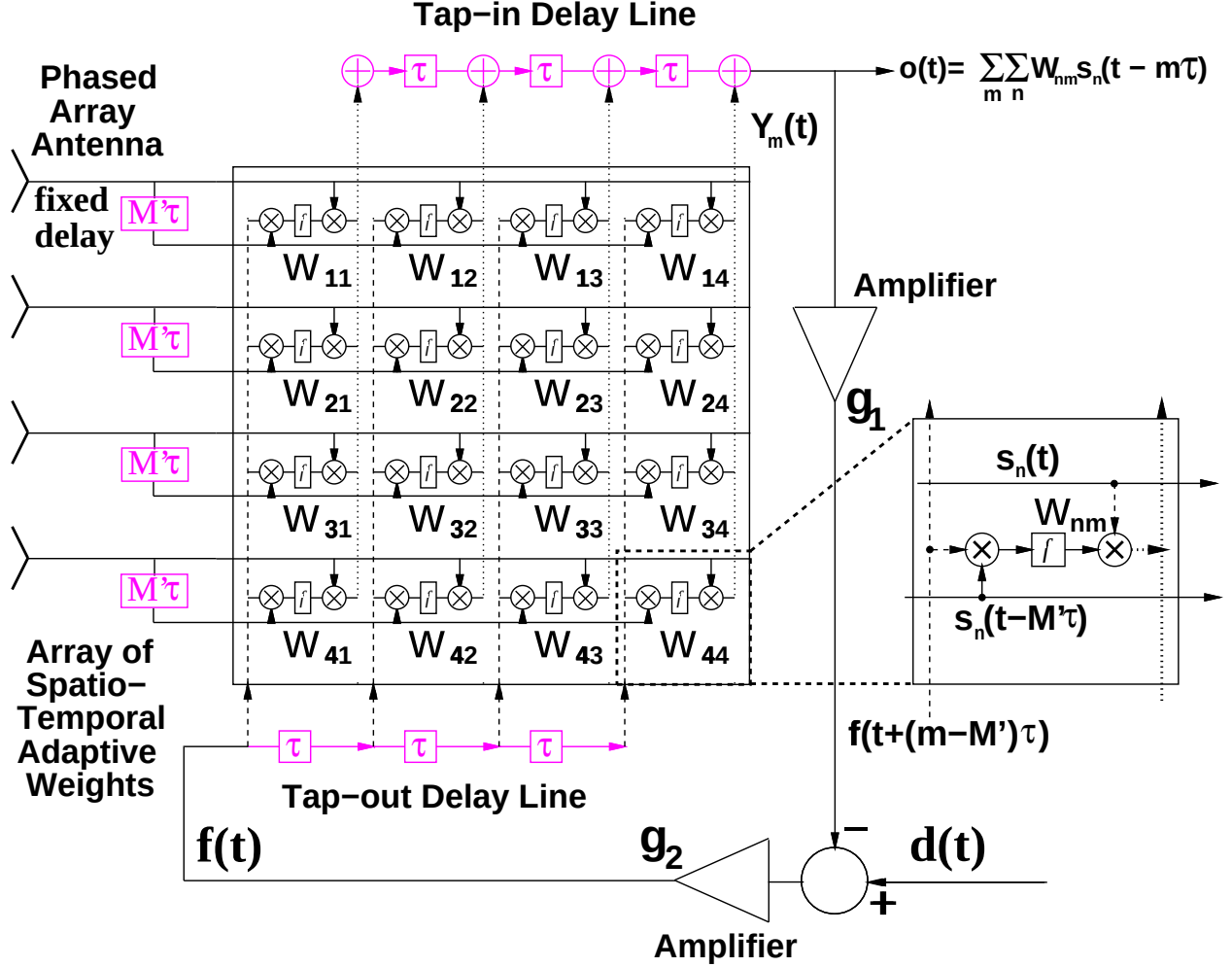


Figure 3: BEAMTAP algorithm for broadband squint-free true-time-delay beamforming using a single output tap-in delay line. For adaptive calculation of the weights within the array, the additional input tapped-delay-line is also required. The desired signal d_t is again subtracted from the output signal o_t to produce the feedback signal f_t .

To operate as an adaptive phased array, the weights must be calculated in response to changes of the signal environment. The LMS algorithm can be readily implemented within the hardware-efficient BEAMTAP beamformer by including an input delay line and a correlator/integrator to calculate each adaptive weight in the $M \times N$ array of weights. Each processing element in the weight matrix array contains an analog multiplier that multiplies the output of the tap-out delay line at that column with the sample from the phased array at that row. These products are locally time-integrated to produce the adaptive weights as the appropriate correlation coefficients

$$W_{nm}^*(t) = \int_{-\infty}^t s_n^*(t_1) f(t_1 + (m - (M - 1))\tau) dt_1. \quad (6)$$

In this expression m varies from 0 to $M - 1$ so that the argument of f at the t limit of integration varies from $t - (M - 1)\tau$ to t and is therefore always causal. These weights are then used

as the coefficients in the subsequent array of multipliers to produce the appropriate beam-steered array output as the scrolling sum of the adaptively-weighted and column-summed contributions:

$$\begin{aligned}
o(t) &= \sum_m Y_m(t - m\tau) \\
&= \sum_m \sum_n \int_{-\infty}^t \delta(t' - [t - m\tau]) s_n(t') W_{nm}^*(t') dt' \\
&= \sum_{n,m} s_n(t - m\tau) \int_{-\infty}^{t-m\tau} s_n^*(t_1) f(t_1 + (m - M')\tau) dt_1 \\
&= \sum_{n,m} s_n(t - m\tau) \int_{-\infty}^t s_n^*(t_2 - m\tau) f(t_2 - M'\tau) dt_2. \tag{7}
\end{aligned}$$

Note that this adaptive algorithm is the exact equivalent of the time-domain LMS adaptive beamformer shown in Eq. 3, except for the simple time shift of the correlation signal f by $M'\tau = (M - 1)\tau$. For correlating against a known reference signal $f(t)$ as required in main beam formation,^{24–26} this time delay is readily accommodated by advancing the reference appropriately. This unwanted delay is more troublesome in correlation-cancellation loop algorithms as required for jammer nulling, in which the feedback signal is given by the desired reference minus the output $f(t) = d(t) - o(t)$, since the output cannot be advanced in time (this would require us to know $o(t)$ before it has been generated). However, we can solve this dilemma by delaying the antenna signals, $s_n(t)$ which are used to write the weights relative to the signals used for reading the weights by an additional $T = M'\tau$

$$\begin{aligned}
o(t) &= \sum_n \sum_m s_n(t - m\tau) \int_{-\infty}^t s_n^*(t_2 - m\tau - M'\tau) f(t_2 - M'\tau) dt_2 \\
&= \sum_n \sum_m s_n(t - m\tau) \int_{-\infty}^{t-M'\tau} s_n^*(t_2 - m\tau) [d(t_2) - o(t_2)] dt_2. \tag{8}
\end{aligned}$$

This produces the correct relative delay between $s_n(t)$ and $o(t)$ inside the weight integral, and $d(t)$ can be delayed as desired. The only difference with Eq. 3 is the upper limit of the integral. This means that the weights are being updated/adapted by the delayed product of $s_n(t)$ and $f(t)$ which will be inconsequential for realistic slow adaptation rates or at steady-state operation. It should also be noted that while this approach does require a second version of each antenna signal, $s_n(t)$, delayed by a fixed amount $M'\tau$, it does not require a tapped-delay-line version of these signals.

A note on the convergence properties of this algorithm are in order, since conventional LMS array processing using DSP techniques are known to only converge for a limited feedback gain.²⁷ The real-time array adaptation in BEAMTAP is a continuous time adaptive algorithm since the weights evolve continuously even though the output waveform may be sampled by the quantized charge packets of an output discrete shift register (the tap-in delay-line) and the array dimension is always sampled. However, since the adaptive weights are updated continuously and are governed by a first order differential equation that is always guaranteed to converge, increasing the feedback gain will not lead to oscillations and unbounded weight growth as in sampled adaptation. Instead, it will simply increase the

adaptation speed and increase the null depths.²⁷ It is important to note though, that noise in the system and unwanted feedthrough of the writing beam onto the velocity matched traveling detector array can lead to loop oscillation and will present a practical limit on the speed of adaptation.

2.2.1 Optical BEAMTAP

The BEAMTAP beamforming algorithm is especially well-suited to a photonic implementation of adaptive beamforming and jammer nulling signal processing in very large phased arrays. The optical approach allows for a huge number of adaptive weights to be stored in the volume of a photorefractive crystal, thereby enabling the broadband processing of very large arrays as the analog multiplication, integration, and final multiplication needed to write and read out the weight matrix is accomplished through the interference, grating formation, and subsequent diffraction of light within the crystal itself. Furthermore, the corresponding digital processing load that would be required to solve these large, broadband, adaptive array signal processing problems for a 1 GHz bandwidth broadband array with 1000 elements and 1000 taps per element exceeds 10^{15} multiply-accumulate operations per second just for the weight readout. The adaptation using LMS is at least as computationally intensive, while covariance matrix estimation and inversion techniques typically utilized in digital adaptive array processing (in the case of the covariance matrix having $10^6 \times 10^6$ elements) is prohibitive since at least 10^{12} operations are required to perturb an already adapted weight matrix—and this must be done at the desired rate of adaptation (typically ms to μ s). Thus, restrictive assumptions on the number of signals are often made to decrease the digital computation load (allowing for more efficient subspace adaptivity); or alternatively, sub-array techniques with smaller numbers of adaptive degrees of freedom are used so that acceptable performance can be obtained unless the number of signals becomes too large. But unless the problem is simplified, these digital computation loads exceed even the most optimistic performance projections for massively-parallel, wafer-scale, and multi-chip-module digital signal processing implementations, making this an appropriate problem for an optical signal processing solution. In addition, RF photonic techniques allow for wide instantaneous bandwidths of 1-2 GHz, limited only by acousto-optic technology, which can be tuned over the full 1-20 GHz RF band using only a single additional electro-optic modulator (EOM) with no mixers or downconverters. Fully photonic processing with only one traveling wave detector avoids noise contributions from the N detectors often required in parallel photonic link systems, thereby potentially improving system dynamic range. In conclusion, therefore, the RF photonic techniques presented here appear to be the only viable solution to these next-generation fully adaptive, large, wideband array processing systems.

3 Optical Systems Analysis

In this section we will perform a detailed analysis of the BEAMTAP optical system. The analysis includes many of the necessary complexities of double-sideband amplitude modulation in EO devices, single-sideband polarization switching diffraction in the AO device, finite response time of the photorefractive crystals, read-write multiplexing and isolation

using polarization and angle multiplexing, as well as spatial filtering of the diffracted orders, interferometric detection on the traveling fringes detector, and the time-delays of the feedback loop. This analysis not only validates the idealized algorithmic analysis presented in 2.2, but also provides valuable information on the necessity of proper alignment of the interferometric beams, the unavoidable bias buildup if a TDI CCD is used for the tap-in delay line, the difficulty with low modulation depths of the EO devices, and the processor bandwidth limitation due to feedback delay²⁸ and its mitigation with the compensating delay of the writing beams. However, the analysis does neglect diffraction, laser drift, and noise (RIN, shot, thermal, etc.) for the sake of simplicity. Numerical simulations of the operation of the processor are then presented that demonstrate several of the compensation mechanisms that are intrinsic to the adaptive holographic weight matrix.

3.1 System Overview

Figure 4 shows one possible optical architecture for the implementation of the BEAMTAP system with the photorefractive crystal in the image plane. (Alternative systems could use the photorefractive crystal in a Fourier or Fresnel plane of either arm; the undiffracted beam from the AO could be used as a reference; or an alternative read-write multiplexing scheme could be utilized²⁹—the key requirement is that the AO is imaged and velocity matched onto the traveling fringes detector.) In the illustrated system a single powerful low noise laser is split by a beamsplitter (amplitude reflectance a_r) and frequency shifted by an EO modulator to within 1 GHz of the RF center frequency (typically ranging between 1-20 GHz). This frequency shifted beam is then distributed through a fiber feed network and is modulated by EOMs with the signals $s_n(t)$ coming from each of the elements of the phased array antenna. The first frequency shifting EOM in combination with the EOMs in the phased array operating at the RF center frequency produce a frequency shift commensurate with that produced by the AOD, allowing for processing of RF signals anywhere within the full 1-20 GHz RF band. The RF frequency domain for double-sideband modulation is illustrated in Fig. 5, although only one sideband will be used in the analysis. Each of the RF modulated signals is then transmitted from the modulators in the phased array through the fiber feed network to an array of terminated fibers. The terminated fibers can be in any topological arrangement (1-D linear array, 2-D hexagonal array, random, etc.) and with an arbitrary permutation with respect to the spatial topology of the phased array, but for simplicity, in this analysis we will assume that there is a 1-to-1 mapping between an equi-spaced linear phased array antenna and an equi-spaced array of optical fibers. The modulated signals from the fiber feed network are then imaged into the photorefractive crystal using the 4f system L_0 .

The transmitted beam from the first beamsplitter (amplitude transmittance $a_t = \sqrt{1 - a_r^2}$) is split into an interferometric reference beam (amplitude $a_t a'_r E_o$) and an input to the acousto-optic Bragg cell or AOD (amplitude $a_t a'_t E_o$). The beam is expanded, collimated, and tilted to the Bragg angle, whereby a feedback signal $f(t)$ applied to the AOD, launches a propagating acoustic wave which modulates the incoming light via the acousto-optic effect and diffracts it at the Bragg angle. Tangential matching in an anisotropic polarization switching device is used here so that the diffracted beam has an orthogonal polarization to

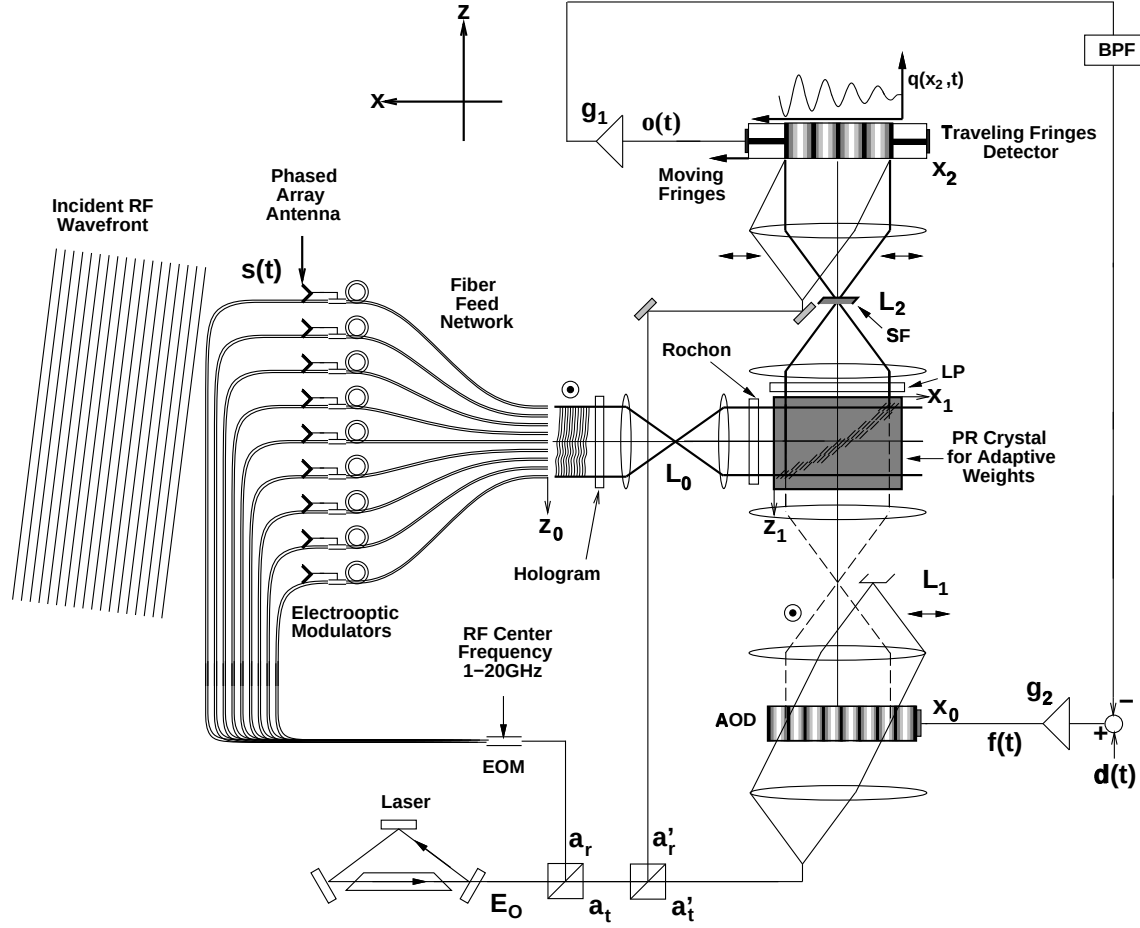


Figure 4: Optical implementation of BEAMTAP. A single coherent laser is split using two beamsplitters with amplitude reflectances a_r and a'_r , and amplitude transmittances a_t and a'_t respectively, to drive both the fiber feed network and the BEAMTAP processor—the fiber feed network from the phased array is shown on the left. The acousto-optic deflector (AOD) (located at plane x_0) interferes with signals from the array (plane z_0) which are imaged through lens system L_0 to form gratings in the photorefractive crystal (PR crystal). Diffraction of the phased array signals off this grating is detected by a synchronous traveling fringes detector (plane x_2), which has a carrier velocity matched to the magnified acoustic velocity of the AOD by the lens systems L_1 and L_2 —producing a resonant charge carrier distribution $q(x_2, t)$. The output signal $o(t)$ is amplified by g_1 , subtracted from the desired signal $d(t)$, and fed back into the AOD as the feedback signal $f(t)$ amplified by g_2 , to close the adaptive feedback loop necessary for the system to cancel any jamming signals present in the signal environment. A Rochon prism, linear polarizer (LP), and spatial filter (SF) are used for the read-write isolation of the AOD beam from the diffracted phased array signals off the grating. The illustrated system places the photorefractive crystal in the image plane of the fiber feed and the AOD, and utilizes orthogonally propagating fields, for illustrative purposes.

that of the undiffracted beam. The diffracted beam is then imaged into the photorefractive crystal using the 4f lens system L_1 , while the undiffracted beam is blocked in the Fourier plane of the lens system L_1 using a spatial filter. The diffracted beam from the AOD interferes with the spatio-temporally modulated optical field launched by the fiber feed network, and the time integration of this interference pattern produces 45° gratings at positions corresponding to delays where the beam containing the desired signal $d(t)$ is well correlated with one of the beams from the fibers containing the signals from the antenna array $s_n(t)$.

Light from the fiber feed network then diffracts off the gratings in the crystal. The time delays (both due to the inherent delays produced by the phased array antenna, and random time delays introduced by the small length mismatches between the fibers) are transformed into diffracted beams at spatial positions proportional to their respective time delays. These position dependent time delays will be delayed and summed with appropriate time alignment by detection with a time-integrating traveling wave detector array matched to the velocity of the image of the AOD. Read-write multiplexing using angle and polarization encoding is implemented using a quarter wave plate, Rochon prism, polarizer, and spatial filter in order to isolate the diffracted readout beam of the phased array from the diffracted write beam of the AOD. Details of read-write multiplexing architectures have been covered in previous publications,^{16,30} so are only briefly delineated in this paper.

The diffracted signals are then imaged onto a time-integrating traveling wave detector implemented with a traveling fringes detector^{20,21} where they interfere with the reference beam from the laser—which must be at the exact angle that the undiffracted DC beam from the AOD would have occupied if it had not been blocked. At each resolvable temporal increment, the traveling fringes detector is read out after the corresponding charge packet has traversed the length of the detector, accumulating and delaying detected signal contributions from each location. The resulting output waveform is an adaptively weighted and coherently reconstructed version of the signal.¹⁷ For closed loop adaptive processing, this output signal $o(t)$ is then subtracted from our desired signal $d(t)$, which is well correlated to the signal of interest $s(t)$ (for instance a known chirp can be broadcast along with an unknown PN waveform). This difference signal generates the feedback signal $f(t) = d(t) - o(t)$, which is then fed back into the AOD. The feedback loop provides the necessary error driven adaptation required to beam steer the main beam towards the desired signal source and null out any undesired jammers present in the signal environment at the input of the phased array antenna—which can include both near-field and far-field, as well as narrowband and broadband jammers.

3.2 The AOD

The single-sideband feedback signal amplified and applied to the acousto-optic device transducer consists of the difference between the processor output signal $o(t)$ and the steering RF signal $d(t)$ such that $f(t) = g_2[\tilde{d}(t - t_d) - g_1 \tilde{o}(t - t_f)]$ (where g_1 and g_2 are the gains of the photodetector output and AOD power amplifier respectively, t_d is a constant reference delay which accounts for the arbitrary timing of the desired signal to ensure that the correlation peak with the arriving array signals is within the AOD aperture, and t_f is the feedback delay due to propagation through wires, amplifiers, and the bandpass filter). The signal is

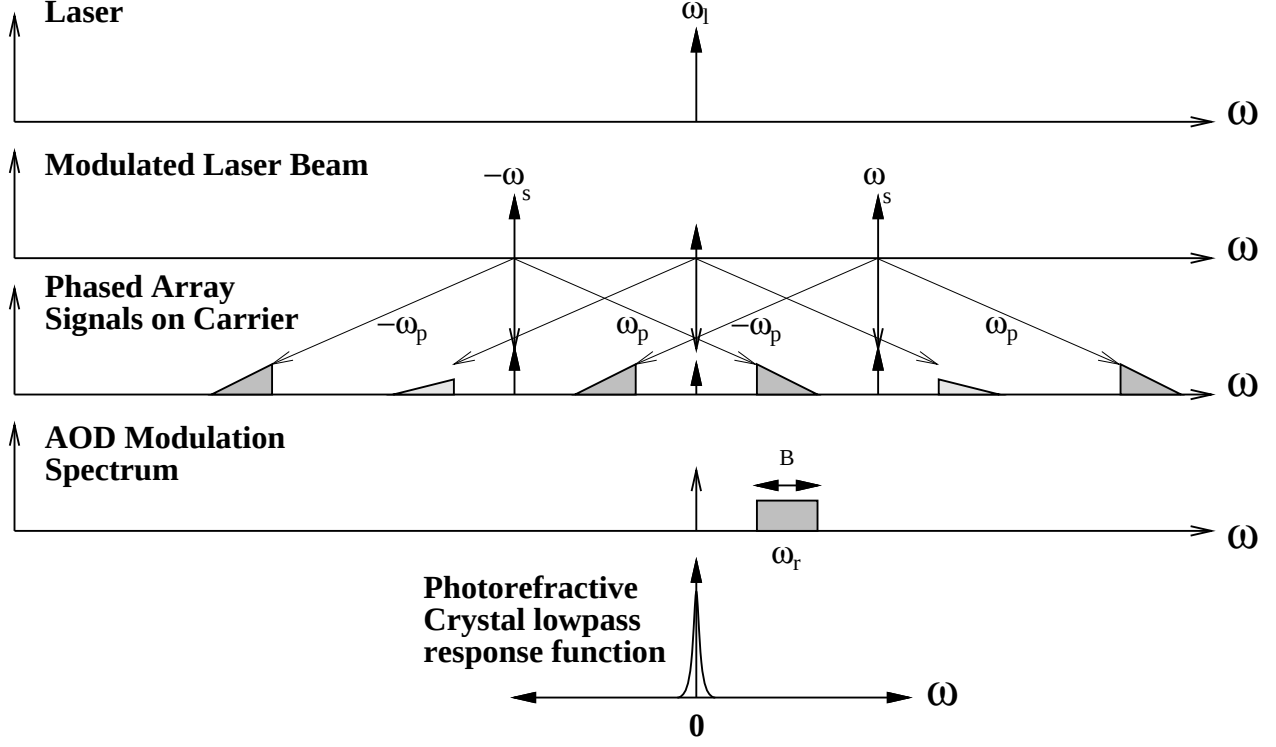


Figure 5: Frequency offset scheme for the phased array radar processor that allows tuning of the processing bandwidth B (determined by the AOD bandwidth and typically limited to 1-2GHz) anywhere within the RF spectrum within the bandwidths of the EOMs. The laser beam ω_l is premodulated by $\pm\omega_s$ (only the negative sideband is used), then the RF signals modulate within a bandwidth of B and are offset by $\pm\omega_p$ such that the desired mix term overlaps the bandwidth B and carrier ω_r of the AOD. The photorefractive crystal only responds to the near DC interferometric products between the phased array signals and AO diffractions, and all moving gratings wash out.

octave bandwidth limited around an RF center frequency ω_r which is implicitly included in the analytic signal representation f_0 , $\tilde{f}(t) = f_0(t) e^{-i\omega_r t}$, where $f_0(t)$ is a real bandlimited baseband signal. The signal will slide through the AOD at the acoustic velocity v_A in the positive x -direction at a plane x_0 and time t_0 , resulting in a modulated output

$$\tilde{f}(x_0, t_0) = g_2 \left[\tilde{d}\left(t_0 - t_d - \frac{x_0}{v_A} - t_{r1}\right) - g_1 \tilde{o}\left(t_0 - t_f - \frac{x_0}{v_A} - t_{r1}\right) \right] \equiv \tilde{f}\left(t_0 - \frac{x_0}{v_A} - t_{r1}\right), \quad (9)$$

where t_{r1} allows for the AOD aperture to be centered in the optical system.

A collimated optical beam with a laser angular radian carrier frequency ω_l which is incident on the AOD at the Bragg angle $\theta_B = \sin^{-1} |\vec{k}_B|/|\vec{k}_0|$, where $|\vec{k}_0| = \omega_l/c = 2\pi/\lambda_0$ is the free space wave-number and $|\vec{k}_B| = \omega_A/c$ is the acoustic wave-number, will diffract off the acoustic wave to produce a diffracted and undiffracted beam

$$E_A(x_0, t_0) = a_t a'_t E_o e^{i(k_B x_0 - \omega_l t_0)} \left[\sqrt{1 - \eta_{AO}^2 |\tilde{f}|^2} \hat{p}_x + \eta_{AO} \tilde{f}\left(t_0 - \frac{x_0}{v_A} - t_{r1}\right) \hat{p}_y \right], \quad (10)$$

where $a_t = \sqrt{1 - a_r^2}$ is the amplitude transmittance of the light from the beamsplitter, η_{AO} is the amplitude diffraction efficiency of the AOD per volt, and $|\tilde{f}(t_0 - \frac{x_0}{v_A} - t_{r1})|^2$ has been written as $|\tilde{f}|^2$ in the undiffracted beam. Anisotropic diffraction in the AOD results in the diffracted and undiffracted beams having orthogonal polarizations, $\hat{p}_y$ and $\hat{p}_x$, respectively. The undiffracted beam can be utilized as an interferometric reference that is automatically aligned at the correct angle to produce a non-dispersive traveling fringe pattern; however, since the strong undiffracted beam will cause erasure and decrease the modulation depth—and resulting diffraction efficiency of the grating—in the photorefractive crystal when used in the image plane, and since the undiffracted beam may have too much residual modulation due to its nonlinear dependence on the signal amplitude, this necessitates the use of a clean co-aligned reference beam that can be re-injected at the image of the DC spot in the Fourier plane after the crystal, as shown in Fig. 4.

The diffracted and undiffracted beams are Fourier transformed by the first lens so the undiffracted beam can be blocked. The diffracted beam is then inverse transformed by the second lens to produce an image of the AOD in the crystal. Noting that $m_1 = \frac{x_1}{x_0}$ is the magnification of lens system L_1 , the electric field distribution can be approximated within the photorefractive crystal. For the sake of simplicity, the effects due to diffraction of the propagating fields are neglected in this analysis. Instead, we will account for propagation with a phase accumulation so that the field within the photorefractive crystal is

$$E_A(x, z, t_0) = a_t a'_t E_0 e^{i(k_B x - \omega t_0)} \eta_{AO} \tilde{f}(t_0 - \frac{x}{v} - t_{r1}) e^{ikz} \hat{p}_y, \quad (11)$$

where $k = \frac{2\pi}{\lambda_o} n$ is the wave-vector length in the photorefractive crystal of index n , and the velocity v_a has been scaled by the magnification to a velocity v . A detailed analysis showing that the effects due to diffraction are holographically compensated for by the processor has already been proven.³¹

3.3 The Phased Array

Next, consider the fiber optic modulation topology shown in Fig. 6. A desired signal produced by a broadband far-field plane wave arriving upon a linear equally spaced array at some angle θ_s is illustrated. The signal at a particular element n will have a differential time delay between adjacent phased array elements $t_p = \frac{d}{c} \sin \theta_s$, where d is the spacing between the elements and c is the speed of light. The desired signal at the n^{th} element at an equi-spaced array can thus be described as $\tilde{s}_n^d = \tilde{r}(t - n t_p)$. Any far-field jammers present within the signal environment will also experience incremental time delays given by $t_j = \frac{d}{c} \sin \theta_j$. If more than one jammer is present, the jamming signal at the n^{th} element can be described as $\tilde{s}_n^j = \sum_j \tilde{j}_j(t - n t_j)$. We will assume that the desired signal and any jamming signals are in a frequency band no greater than the bandwidth of the AOD centered around ω_p , but that this center frequency can be anywhere within the modulation bandwidth of the EOMs. A single sideband of the signal detected by the n^{th} array element is thus

$$\tilde{s}_n(t) = \tilde{r}(t - n t_p) + \sum_j \tilde{j}_j(t - n t_j), \quad (12)$$

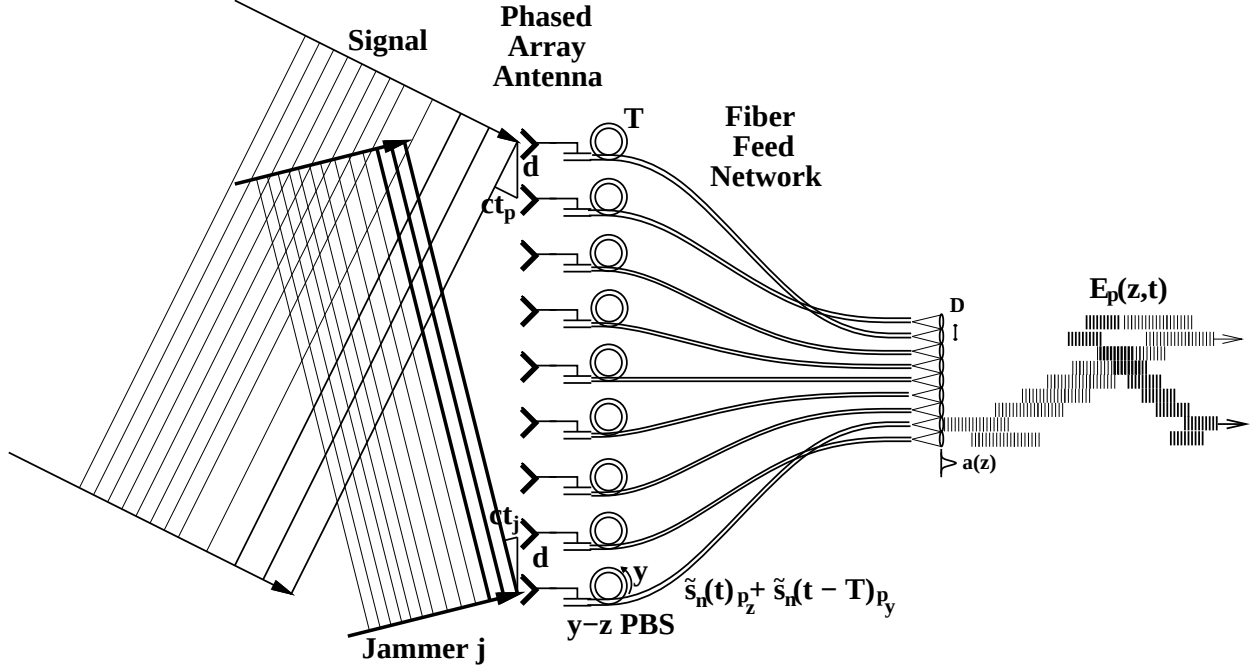


Figure 6: An example of a linear equi-spaced phased array and a random fiber optic feed network. A broadband RF signal is incident from one angle and a jammer j from another angle onto a phased array antenna with element spacing d , creating time delays t_p and t_j between the elements of the array for the signal and jammer, respectively. Both signals are modulated onto a much higher frequency optical carrier and transduced into the optical domain; then these signals are propagated through an optical feed manifold to a linear array of polished fiber ends. While propagating through the fiber, each beam passes through a polarizing beam splitter so that the $\hat{p}_y$ polarized writing beam passes through the delay loop and experiences a time delay T with respect to the $\hat{p}_z$ polarized reading beam. Typical fiber cores of $8 \mu\text{m}$ and spacings of $250 \mu\text{m}$ are shown being collimated by a lens-let array, which allows us to make the simplifying assumption of propagation without diffraction used in the analysis of this section. These lens-lets are not required in the real system, and the simplifying assumption of propagation without diffraction is not required in practice, but is illustrated here for analytic and diagrammatic simplicity.

where the carrier term ω_p is implicit within the single-sideband analytic representations $\tilde{r}(t)$ and $\tilde{j}(t)$. More generally, the nt_p and/or the nt_j term can instead be an arbitrary function of n to represent any random time-delays caused by positional errors in the placement of the array elements, conformal arrays, near-field signals/jammers, or an arbitrary spatial permutation of the topology of the array of fibers with respect to the array elements.

Amplitude modulation by the first frequency shifting EOM before the phased array offsets the light carrier ω_l by $\pm\omega_s$ giving a new carrier $\omega_c = \omega_l \pm \omega_s$ (where we will choose to use only one of the sidebands and reject the other with a fiber bandpass filter). After being amplitude modulated by the N array of signals at the fiber feed network, the electric field

distribution launched by the fibers is

$$E_P(z_0, t_0) = \frac{a_r}{\sqrt{N}} E_0 \sum_{n=1}^N e^{-i[\omega_c(t_0 - t_n)]} a(z_0 - nD_0) \times \left[\sqrt{1 - \eta_p^2 |\tilde{s}_n|^2} + \eta_p \tilde{s}_n(t_0 - t_n - T) + \eta_p^* \tilde{s}_n^*(t_0 - t_n - T) \right] \hat{p}_y, \quad (13)$$

where $\frac{a_r}{\sqrt{N}}$ is the amplitude of the light after equally splitting to each of the N fibers, $a(z_0 - nD_0)$ is the aperture function of the n^{th} fiber (typically nearly Gaussian of width $5 - 9\mu\text{m}$) at position nD_0 , where D_0 is the spacing between the fibers (for v-grooves the spacing $D_0 = 250\mu\text{m}$), η_p is the amplitude modulation efficiency of the EOMs per volt (assumed to be the same for all modulators), $|\tilde{s}_n|^2 \equiv |\tilde{r}(t_0 - n t_p - t_n - T) + \sum_j \tilde{j}_j(t_0 - n t_j - t_n - T)|^2$, t_n is a random time delay which has been introduced to account for phase and time shifts due to the fiber feeds not being cut to exactly the same length—which affects both the light carrier ω_c (even for sub micron length mismatches) and the phased array RF signal at ω_p (only for length mismatches on the scale of centimeter RF wavelengths), and T is the round-trip feedback loop time delay which, as described in Sec. 2.2, is necessary to ensure causality of the final output signal $o(t)$. This time delay is implemented on each phased array signal by using a fiber polarizing beamsplitter in each fiber; this causes the $\hat{p}_y$ polarized writing beam to pass through a fiber delay loop, while the $\tilde{p}_z$ polarized reading beam propagates directly through the fiber feed network. Thus Eq. 13 describes the electric field distribution of the writing beam while the reading beam will be $\tilde{p}_z$ polarized without the delay T .

It is vital to realize that these lengths can easily be perturbed by small motions of the fibers or temperature changes, which give rise to t_n , and although such fluctuations will have no effect on the RF carrier phase, the optical phase can swing wildly and must be adapted for. In the system presented in this paper adaptation will occur in a time well below a millisecond (much faster than the photorefractive hologram response time¹⁶), which should be sufficient to compensate for these phase fluctuations. The polarization preserving fiber feed produces an output having the same polarization as the diffracted beam from the AOD, $\hat{p}_y$. We are representing the EOM as being operated as an amplitude modulator, but for the sake of simplicity the complex conjugate term will be subsequently dropped to produce a single-sideband phase modulator. Ideally, we would utilize single-sideband suppressed carrier modulators which would eliminate the DC term as well. Including these terms in the analysis yields essentially the same overall result since they are at the wrong frequency to record stationary gratings and thus will not affect the PR crystal except through incoherent erasure. In addition, the phase in the $\hat{p}_y$ polarized fiber delay loops must be stabilized with respect to the undelayed $\tilde{p}_z$ polarized phases. Although this can be accomplished with individual polarization locking feedback to loop fiber stretchers, a single novelty filter³² can be used instead to stabilize all of the fiber loops simultaneously (and to remove the unwanted DC term as well). We leave the details of this loop stabilization to a subsequent paper, and henceforth will assume a phase stable delay by T in all of the fiber loops.

This phased array signal beam is then imaged through lens system L_0 , and noting that $m_0 = \frac{z_1}{z_0} = \frac{D_1}{D_0}$, the electric field distribution within the photorefractive crystal can be described by including a propagation term that represents phase accumulation (in the negative

x-direction)

$$E_P(x, z, t_0) = \frac{a_r}{\sqrt{N}} E_0 \sum_{n=1}^N e^{-i[\omega_l(t_0-t_n)]} a(z - nD) \left[\sqrt{1 - \eta_p^2 |\tilde{s}_n|^2} + \eta_p \tilde{s}_n(t_0 - t_n - T) \right] e^{-ikx} \hat{p}_y, \quad (14)$$

where the offset frequency of ω_s has now been incorporated implicitly into $\tilde{s}_n(t_0)$. We will choose the modulation frequencies such that $\omega_r = \omega_s + \omega_p$, so the resulting modulation produced by the EOMs overlaps with the Doppler frequencies produced by the AOD, as shown previously in Fig. 5.

3.4 Formation of the Grating

As the optical beam from the AOD interacts with the beam from the phased array, and if the frequency spectra of the two beams overlap, an index grating is formed within the photorefractive crystal due to the stationary interference pattern. Using first order analysis, the grating evolution can be approximated by

$$G(x, z, t') = \beta \int_{-\infty}^{t'} E_A(x, z, t_0) E_P^*(x, z, t_0) e^{-\frac{(t'-t_0)}{\tau}} dt_0 + cc, \quad (15)$$

where β is the sensitivity of the crystal (cm^2/J), τ is the photorefractive time constant, and the details of the photorefractive dynamics have been idealized from their much more complex form.³³ For times $t' \gg \tau$ the convolution with $e^{-\frac{t'-t_0}{\tau}}$ results in a causal Lorentzian lowpass filter in the temporal frequency domain with a width $\Delta f = \frac{1}{2\pi\tau}$, typically in the range of Hz-kHz. Thus only stationary gratings can be written in the photorefractive crystal between equally Doppler frequency shifted beams, which are then weighted by the photorefractive time constant τ , which in turn is inversely proportional to the DC intensity level $\tau \propto \frac{\tau_o}{I_{DC}}$, where $I_{DC} = |E_A(x, z, t)|^2 + |E_P(x, z, t)|^2$. The spatial frequency response of the photorefractive crystal selects only high spatial frequency gratings to be recorded, although DC terms do lead to grating erasure.³⁴ Thus the photorefractive grating is given by

$$G(x, z, t') = \beta \int_{-\infty}^{t'} \left[e^{ik(z+x)} e^{ik_B x} \frac{a_r a_t a'_t}{\sqrt{N}} E_0^2 \eta_{AO} \tilde{f}(t_0 - \frac{x}{v} - t_{r1}) \right] \times \left[\sum_{n=1}^N e^{-i\omega_l t_n} a^*(z - nD) \left(\sqrt{1 - \eta_p^2 |\tilde{s}_n|^2} + \eta_p \tilde{s}_n^*(t_0 - t_n - T) \right) \hat{p}_y \right] e^{-\frac{(t'-t_0)}{\tau}} dt_0. \quad (16)$$

Since the un-modulated EOM beam produces a moving grating with the diffracted AOD beam at the RF carrier frequency ω_r , this grating washes out due to the lowpass nature of the photorefractive crystal, so

$$G(x, z, t') = e^{ik(z+x)} e^{ik_B x} \kappa_0 \sum_{n=1}^N e^{-i\omega_l t_n} a^*(z - nD) \int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x}{v} - t_{r1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0, \quad (17)$$

where $\kappa_0 = \beta E_0^2 \tau \frac{a_r a_t a'_t}{\sqrt{N}} \eta_A \eta_P$.

The phased array signal environment consists of the RF signal of interest $\tilde{r}(t)$ and various jammers $\tilde{j}_j(t)$. Therefore the desired signal $\tilde{d}(t)$ is chosen (or generated adaptively²⁷) to have good correlation properties with $\tilde{r}(t)$ and poor correlation with the jammers so that a cross-correlation is formed between the feedback signal $\tilde{f}(t_0 - \frac{x}{v} - t_{r1})$ (which contains the desired signal $\tilde{d}(t_0 - t_d - \frac{x}{v} - t_{r1})$) and $\tilde{s}_n^*(t_0 - t_n - T)$, the signals from each element of the phased array. The cross-correlation results in 45° grating-lets, corresponding to each resolvable time delay between adjacent antenna elements, as indicated by the $e^{ik(z+x)}$ term wherever $\tilde{d}(t_0 - t_d - \frac{x}{v} - t_{r1})$ overlaps with $\tilde{s}_n^*(t_0 - t_n - T)$ within the crystal. The gratings, whose amplitudes correspond to a weight magnitude, are written with a complex phase factor (corresponding to the phase of the complex weights and are physically manifested as a shift of the high frequency grating) that accounts for optical propagation delays and RF signal delays at positions corresponding to those respective time delays. Assuming the time delay distortions of the fiber feed network are small (the variations of t_n over n are less than $1/\text{BW}$), the envelope of the 45° grating-lets will lie on a tilted line, whose tilt angle corresponds to a specific angle of arrival. Overall time delays can be accounted for by shifts of this line, and changes in the angle of arrival are manifested as changes of the tilt angle of the envelope of the gratings. The gratings within the cross-correlation envelope evolve as a function of time, allowing the processor to adapt to changing RF signal environments as well as slow (kHz or slower) phase changes due to temperature or mechanical fluctuations within the fiber feed network. Small phase drifts simply shift the phase of the gratings within the envelope, while larger changes of the random fiber delays will shift the position of the envelope of the gratings.

3.5 Diffraction of the Phased Array Signal Beam off the Grating

As light from each of the phased array signal fibers propagates through the crystal, picking up a phase factor of e^{-ikx} , each beam of light interacts with the local gratings and diffracts off of it. An additional phase of $e^{ik(L-z)}$, where L is the length of the crystal, is accumulated as each of the diffracted beams propagate along z to the face of the crystal. As a result, the diffracted beam at the edge of the crystal can be calculated by

$$\begin{aligned}
E_d(x_1, t') &= \int_0^L E_P(x, z, t') G(x, z, t') e^{ik(L-z)} dz \\
&= \int_0^L \frac{b_1}{\sqrt{N}} E_0 \sum_{n'=1}^N e^{-i[\omega_l(t'-t_{n'})]} a(z - n'D) \left[\sqrt{1 - \eta_p^2 |\tilde{s}_{n'}|^2} + \eta_p \tilde{s}_{n'}(t' - t_{n'}) \right] e^{-ikx} \tilde{p}_z \times \\
&\quad e^{ik(z+x)} e^{ik_B x} \kappa \sum_{n=1}^N e^{-i\omega_l t_n} a^*(z - nD) \times \\
&\quad \int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x}{v} - t_{r1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 e^{ik(L-z)} dz,
\end{aligned} \tag{18}$$

where we have use the expression for the undelayed version of the phased array field amplitude to diffract off the grating. Note that the phase accumulated by propagating in the negative

x -direction e^{-ikx} is cancelled with the x dependence of the phase within the grating $e^{ik(z+x)}$; and at each z the phase accumulated by propagating to the output $e^{ik(L-z)}$ after diffracting off the grating cancels with the z dependence of the grating $e^{ik(z+x)}$. As a result, each of the components of light which diffracts off the grating from the fibers picks up the same overall phase e^{jkL} , and therefore adds coherently. Because the fiber profiles do not overlap the undiffracting fields are spatially orthogonal to each other so that $n = n'$ and the two sums collapse, which allows the random phase delays present in the carrier $e^{i\omega_1 t_{n'}}$ to be canceled with the holographically encoded phase $e^{-i\omega_1 t_n}$.

The signal strength received from the phased array antennas will usually be small and electrical pre-amplification may be required at each element of the antenna before optical modulation, but even with amplification the modulation depth of the EOM is typically $\ll 1\%$. Thus there are 2 consequences of the DC term: erasure of the desired grating—thereby decreasing the grating strength (and therefore the null depth); and unwanted diffraction of the DC term from each fiber off of the correlation grating from that fiber—thereby producing unwanted bias when detected on the TFD. Thus the DC term in E_p which diffracts off the grating (although with an incorrect phase front due to the fiber time delays) could potentially be much stronger than the modulated term, creating such a strong bias that it could swamp out the signal of interest. The actual diffraction efficiency of the unmodulated DC terms from the phased array off the gratings will depend on the details of the time delays t_n , and due to coherent superposition of the diffractions off different parts of the gratings, the diffraction efficiency will tend to be zero for large array sizes. But in order to avoid any unwanted diffraction of the strong DC terms, the EOMs should use a suppressed carrier modulation scheme³⁵ (or a photorefractive novelty filter could be used on the phased array feed fiber outputs to remove the DC carrier term³²). We will thus assume that the DC term is not present in E_p . Finally, noting that $\int_0^L |a(z - nD)|^2 dz = 1$ is just the total normalized optical power present in the n^{th} fiber,

$$E_d(x_1, t') = e^{i(k_B x_1 - \omega_1 t')} e^{ikL} E_o \kappa_1 \sum_{n=1}^N \tilde{s}_n(t' - t_n) \int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x_1}{v} - t_{r1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 \tilde{p}_z, \quad (19)$$

where $\kappa_1 = \kappa_0 a_r \eta_p$. Equation 19 shows that the diffracted output from the photorefractive crystal convolves the cross-correlation between the feedback signal and the detected signal with the signals from the phased array antenna, with the detected signal used in the cross-correlation being delayed by the round-trip delay time T to ensure causality. Note that the random phase factors which were originally present in the beam have been holographically compensated for, and have been removed from the diffracted signals. Over a region defined by the transverse width of the grating envelope we will see a traveling wave version of the signals from the various elements of the phased array propagating at a velocity v independent of the arrival angle of the signal.

3.6 Read-Write Isolation

As described so far, the readout diffracted beam from the phased array is not separable in angle from the diffracted write beam from the AOD. For our adaptive phased-array processor,

it is important to be able to separate these two beams so that the writing beam can be completely blocked from the detector plane. Otherwise, any leakage of the writing beam onto the traveling fringes detector will limit the available loop gain, and therefore, the processor null depth.²⁹ This can be accomplished in many ways using polarization, timing, or angle multiplexing—here we consider a scheme that first vertically (i.e. orthogonal to the interaction plane of Bragg selectivity) deflects the undelayed portion of the light from the phased array fiber feed by a Rochon prism (with a small deflection angle θ_r on the order of 1 degree). As discussed previously, the writing and reading beams have orthogonal polarizations ($\hat{p}_y$ and $\tilde{p}_z$, respectively) due to the polarizing beamsplitter used in the fiber feed network to ensure the writing beam propagates through the fiber loop and is delayed by T with respect to the reading beam. The polarization of the undeflected beam $\hat{p}_y$ and deflected beam $\tilde{p}_z$ from the Rochon prism is shown in the k -space diagram in Fig. 7, and are designated as k_{p1} and k_{p2} for the writing and reading beams, respectively. The reading beam is tilted vertically in the direction of Bragg degeneracy, allowing for the light to diffract off the original grating k_G which was written between the two y-polarized beams k_P and k_A , the beams from the phased array and AOD, respectively. This produces a diffracted beam k_d which is vertically tilted from the reference signal beam from the AOD k_A , so that in the Fourier plane of the first lens in lens system L_2 (with focal length F_2) they are spatially separated and the beam k_A that wrote the grating can be blocked.¹⁶ A linear polarizer with the transmission axis in the x -direction is also used to block the write beam from the AOD (which is polarized in the y -direction) to increase the overall isolation ratio between the reading and writing beams. Spatial filtering the beams in the Fourier plane then allows us to fully separate the diffracted phased array beam, which has been vertically shifted by $y_r = \tan \theta_r F_2$, from any leakage through the polarizer of the signals from the AOD. This scheme requires a z -polarized beam to be diffracted into an x -polarized beam by the grating along the xz -direction, which is actually the conventional geometry for high diffraction efficiency readout using BaTiO₃ (either crystallographic or 45° cuts³⁶) or SBN.

At this point, the off-axis interferometric reference beam (at the laser frequency ω_l) needs to be re-injected. Its position should correspond to the position in the Fourier plane (shifted vertically by y_r) where the DC beam from the AOD would have been imaged to if it had not been blocked in the first Fourier plane in lens system L_1 . This can be done by placing a fiber carrying the reference beam with amplitude $a_t a'_r E_0$ at the correct position in the Fourier plane, or using the appropriate pick off mirrors as alluded to in Fig. 4. A final lens can then be used to re-transform the diffracted beam coming from the phased array in the photorefractive crystal, and the injected interferometric reference beam, onto the traveling fringes detector.

3.7 Interferometric Traveling Fringes Detector

Assuming a magnification of the 4f system of m_2 and a re-alignment of the system axis in the vertically tilted plane, the total field at the traveling fringes detector is

$$E_d(x_2, t') = E_0 e^{i(k_B x_2 - \omega_l t')} \left[a_t a'_r + \eta_D \kappa_1 \sum_{n=1}^N \tilde{s}_n(t' - t_n) \times \right.$$

Read-Write Multiplexing Technique

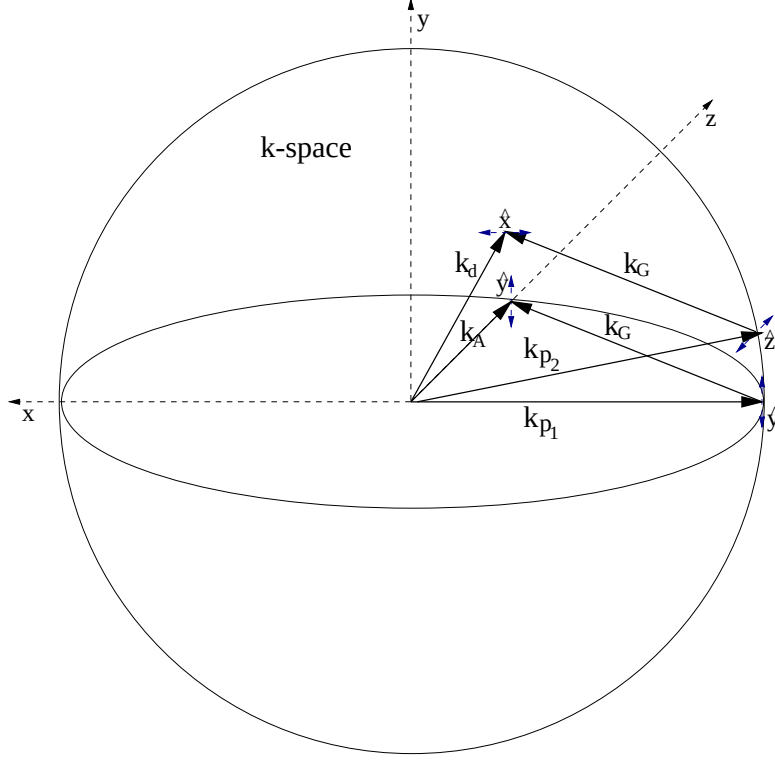


Figure 7: k -space representation of the polarization angle and time multiplexed recording and readout geometry used to separate the modulated phased array beam k_P from the diffracted AOD beam which was used to write the grating. The polarization $\tilde{p}_z$ of the deflected beam k_{p_2} is orthogonal to the polarization $\hat{p}_y$ of the undeflected beam k_{p_1} . The second beam is tilted vertically in the direction of Bragg degeneracy, allowing light to diffract off the grating k_G . This produces a diffracted beam k_d with polarization $\hat{p}_x$ which is vertically tilted from the reference signal beam from the AOD k_A with polarization $\hat{p}_y$.

$$\int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x_2}{v_D} - t_{r_1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 \Big] \hat{p}_x, \quad (20)$$

where $m_2 = \frac{x_2}{x_1}$, η_D is the overall amplitude transmission of the diffracted beams through the read-write multiplexing architecture, v_D is the velocity v scaled by the magnification m_2 so that it matches the detector velocity, and we have dropped the constant phase term e^{ikL} without any loss of generality.

The intensity at the detector is given by

$$I(x_2, t') = (a_t a'_r E_0)^2 + (\eta_D \kappa_1)^2 \left| \sum_{n=1}^N \tilde{s}_n(t' - t_n) \int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x_2}{v_D} - t_{r_1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 \right|^2 + \kappa_2 \sum_{n=1}^N \tilde{s}_n(t' - t_n) \int_{-\infty}^{t'} \tilde{f}(t_0 - \frac{x_2}{v_D} - t_{r_1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 + cc, \quad (21)$$

where $\kappa_2 = a_t a_r' \eta_D \kappa_1$.

A traveling fringes detector is based upon the synchronous drift of photo-generated carriers with a moving interference pattern^{20,21} generated by interfering two beams of light at different frequencies—the diffracted beam from the phased array and the interferometric reference beam, which are also utilized to time delay the RF signals. At a given moment in time, the spatially modulated light which is incident on the photoconductive layer of the detector will generate photocarriers in proportion to the local intensity, and these carriers will drift with a velocity proportional to the applied bias voltage. Varying the applied bias voltage in the photoconductor changes this drift velocity, and a resonance peak occurs when it equals the fringe velocity of the moving interference pattern. For the desired signal diffracted by the grating, the interferometric fringes will all move at the magnified acoustic velocity, which can be set equal to the velocity of the photo-generated carriers by fine tuning the bias voltage. The unwanted but unavoidable diffractions of the jammers off of the sidelobes of the photorefractive grating, however, will not have the necessary frequency-angle relationship to produce constant velocity moving fringes at the carrier and drift velocity. Thus the jammers will be somewhat suppressed and washed out—both by the angular selectivity of the diffraction by the photorefractive crystal, and in the frequency domain by the temporal filtering associated with resonant detection of constant velocity interferometric fringes on the traveling fringes detector.

The first two terms in Eq. 21 are unwanted DC terms, and will contribute only to bias in the detector. The desired detected interference pattern however, is moving at a velocity $v_D = m_1 m_2 v_A$, the acoustic velocity of the AOD scaled by the overall magnification of the lens systems L_1 and L_2 . If the AOD has a time-bandwidth product M , with spatial resolution Δx_2 , then the traveling fringes detector will detect the moving interference pattern over a time $T_D = \frac{M \Delta x_2}{m_1 m_2 v_A} = \frac{M \Delta x_2}{v_D}$ with a responsivity R . Assuming that the magnification has scaled the velocity and been fine tuned with the bias voltage across the device to be synchronous with the photogenerated carriers, the final output is

$$o(t) = R \int_{-T_D/2}^{T_D/2} \int_{-\infty}^t I(x_2, t') \delta[(t' - t) - \frac{x_2}{v_D} + t_{r_2}] dt' dx_2 / v_D = R \int_{-T_D/2}^{T_D/2} I(x_2, t + \tau_x - t_{r_2}) d\tau_x, \quad (22)$$

where t_{r_2} is another reference delay which allows the traveling fringes detector to be centered within the optical system and the limits of integration to remain causal, and $\tau_x = x_2 / v_D$. Substituting this expression into the detected signal of interest,

$$\begin{aligned} o(t) &= R \kappa_2 \int_{-T_D/2}^{T_D/2} \sum_{n=1}^N \tilde{s}_n(t - t_n + \tau_x - t_{r_2}) \int_{-\infty}^{t + \tau_x - t_{r_2}} \tilde{f}(t_0 - \tau_x - t_{r_1}) \tilde{s}_n^*(t_0 - t_n - T) dt_0 d\tau_x \\ &= R \kappa_2 \int_{-T_D/2}^{T_D/2} \sum_{n=1}^N \tilde{s}_n(t - t_n + \tau_x - t_{r_2}) \times \\ &\quad \int_{-\infty}^t \tilde{f}(t_1 - (t_{r_1} + t_{r_2})) \tilde{s}_n^*(t_1 - t_n - T + \tau_x - t_{r_2}) dt_1 d\tau_x. \end{aligned} \quad (23)$$

The feedback signal $\tilde{f}(t_1 - (t_{r_1} + t_{r_2}))$ contains the output signal with a feedback time delay t_f , which when combined with the reference delays $(t_{r_1} + t_{r_2})$, provides the total round-trip time delay $T = t_f + (t_{r_1} + t_{r_2})$. The reference delays were used to center the AOD and traveling fringes detector within the optical system, and $(t_{r_1} + t_{r_2})$ represents the total fixed time delay which is incurred by each of the singals propagating through the two tapped-delay-lines. Expanding the feedback signal into its desired and output signal components,

$$\begin{aligned}
\tilde{o}(t) &= R \kappa_2 \int_{-T_D/2}^{T_D/2} \sum_{n=1}^N \tilde{s}_n(t - t_n + \tau_x - t_{r_2}) \times \\
&\quad \int_{-\infty}^t g_2 \left[\tilde{d}(t_1 - t_d) - g_1 \tilde{o}(t_1 - T) \right] \tilde{s}_n^*(t_1 - t_n - T + \tau_x - t_{r_2}) dt_1 d\tau_x \\
&= R \kappa_2 \int_{-T_D/2}^{T_D/2} \sum_{n=1}^N \tilde{s}_n(t - t_n + \tau_x - t_{r_2}) \times \\
&\quad \int_{-\infty}^{t-T} g_2 \left[\tilde{d}(t_2 + (T - t_d)) - g_1 \tilde{o}(t_2) \right] \tilde{s}_n^*(t_2 - t_n + \tau_x - t_{r_2}) dt_2 d\tau_x.
\end{aligned} \tag{24}$$

Equation 25 shows that with the total round trip delay T appropriately incorporated into both the feedback signal and the writing signal from the phased array, a temporal equation is created which is identical that of Eq. 8. A cross-correlation is formed between fixed delayed versions of the signal from the antenna array and a delayed version of the feedback signal, which is then convolved with the instantaneous signals from the array to provide the final output $\tilde{o}(t)$. Since $\tilde{o}(t)$ appears on both sides of this integral equation it is difficult to solve in the time domain; hence we will susequently transform to a Fourier domain representation.

3.8 Frequency Domain Analysis

Equation 25 shows the adaptation of the output to an arbitrary signal environment with an arbitrary number of jamming signals present, remembering that $\tilde{s}_n(t) = \tilde{r}(t - n t_p) + \sum_j \tilde{j}_j(t - n t_j)$. The solution of this equation can be derived by transforming to the frequency domain and algebraically manipulating the spectra to solve for $\tilde{O}(f)$. At steady-state with $t \gg T$, the output signal is a spatio-temporal convolution of the signal from the phased array with the cross-correlation function of the delayed feedback signal with the delayed version of the phased array signal. When transformed to the temporal Fourier domain Eq. 25 becomes

$$\tilde{O}(f) = R \kappa_2 \sum_n \tilde{S}_n(f) g_2 \left[\tilde{D}(f) e^{i2\pi f(T-t_d)} - g_1 \tilde{O}(f) \right] \tilde{S}_n^*(-f) * T_D \text{sinc}(T_D f), \tag{25}$$

where $*$ denotes a convolution. Solving for the frequency domain output $\tilde{O}(f)$,

$$\tilde{O}(f) = \frac{g_2 R \kappa_2 \sum_n \left| \tilde{S}_n(f) \right|^2 \tilde{D}(f) e^{i2\pi f(T-t_d)} * T_D \text{sinc}(T_D f)}{1 + g_1 g_2 R \kappa_2 \sum_n \left| \tilde{S}_n(f) \right|^2 * T_D \text{sinc}(T_D f)}, \tag{26}$$

where the $e^{i2\pi f(T-t_d)}$ phase term denotes the arbitrary timing necessary to ensure the cross-correlation between the desired signal and the signals from the phased array occurs within the photorefractive crystal. Noting that the Fourier transform of the input signal $\tilde{s}_n(t)$ is $\tilde{S}_n(f) = \tilde{R}(f)e^{-i2\pi n t_p} + \sum_j \tilde{J}_j(f)e^{-i2\pi n t_j}$ and is dominated by the jammers, the term in the denominator will simplify to the sum of the powers of the jamming signals present $\sum_n |\sum_j \tilde{J}_j(f)|^2 = \sum_n \sum_j |\tilde{J}_j(f)|^2$ since the jammers are mutually independent. This is a valid approximation for many signal environments since typically the signal of interest is buried beneath the noise floor, while the jamming signal may be as large as 30 - 60 dB above the noise. The frequency domain sinc function represents the limitation of temporal degrees of freedom, but can be approximated as a delta function when T_D is much greater than the correlation time of the desired signal,³⁷ yielding

$$\tilde{O}(f) = \frac{\frac{g}{g_1} \sum_n \tilde{S}_n(f) [\tilde{S}_n^*(-f) \tilde{D}(f) e^{i2\pi f(T-t_d)}]}{1 + g \sum_n \sum_j |\tilde{J}_j(f)|^2}, \quad (27)$$

where $g = g_1 g_2 R \kappa_2 T_D$ is the net gain around the loop.

Spatial Fourier transformation can now be utilized to illustrate the spatio-temporal frequency response of the system. Noting that $\tilde{S}_n(f) = \sum_l \tilde{S}(k_l, f) e^{i k_l n d}$, and that $\tilde{S}(k_l, f) = \sum_n \tilde{S}_n(f) e^{-i k_l n d}$ where $k_l = l \frac{2\pi}{Nd}$ (for $l = -N/2, \dots, N/2$), and expanding the detected signal into its constituent signal and jammer components at their respective angles of arrival ($\tilde{S}_n(f) = \tilde{R}(f) e^{i2\pi \frac{f}{c} \sin(\theta_r) n d} + \sum_j \tilde{J}_j(f) e^{i2\pi \frac{f}{c} \sin(\theta_j) n d}$ since $t_p = \frac{f}{c} d \sin \theta_r$ and $t_j = \frac{f}{c} d \sin \theta_j$) yields

$$\begin{aligned} \tilde{S}(k_l, f) = & \left[\tilde{R}(f) \delta \left(k_l - \frac{f}{c} \sin(\theta_r) \right) + \right. \\ & \left. \sum_j \tilde{J}_j(f) \delta \left(k_l - \frac{f}{c} \sin(\theta_j) \right) \right] * Nd \text{sinc}(Nd k_l). \end{aligned} \quad (28)$$

Thus broadband signals lie on a tilted locus in spatio-temporal Fourier space (pivoting through $k_l = 0, f = 0$), and are blurred by the finite array size.

The signal of interest $\tilde{R}(f) \delta \left(k_l - \frac{f}{c} \sin \theta_r \right)$ detected by the array which correlates perfectly against the desired signal $\tilde{D}(f)$ can be separated into a component $\alpha \tilde{D}(f)$ which correlates with $\tilde{D}(f)$, and a component $\alpha' \tilde{D}'(f)$ which is independent of it, allowing further simplification of the output since only the part which is correlated with the desired signal will write a grating within the photorefractive crystal. Since the desired signal is also uncorrelated to any of the jamming signals the output takes on the form

$$\tilde{O}(f) = \sum_{k_l} \tilde{S}(k_l, f) \frac{\frac{g}{g_1} \alpha |\tilde{D}(f)|^2 e^{i2\pi f(T-t_d)} \delta \left(k_l - \frac{f}{c} \sin \theta_r \right) * b(k_l, f)}{1 + g \sum_{k_l} \sum_j |\tilde{J}_j(f)|^2 \delta \left(k_l - \frac{f}{c} \sin \theta_j \right) * b(k_l, f)}, \quad (29)$$

where $b(k_l, f) = T_d \text{sinc}(T_d f) Nd \text{sinc}(Nd k_l)$ denotes the total blur function. The linear adaptive array output frequency response is given by a weighted signal spatio-temporal frequency spectrum which is summed across all spatial frequencies

$$\tilde{O}(f) = \sum_{k_l} \tilde{S}(k_l, f) \tilde{T}(k_l, f), \quad (30)$$

and this allows the steady-state spatio-temporal transfer function of the BEAMTAP processor to be defined as

$$\tilde{T}(k_l, f) = \frac{\frac{g}{g_1} \alpha |\tilde{D}(f)|^2 e^{i2\pi f(T-t_d)} \delta\left(k_l - \frac{f}{c} \sin\theta_r\right) * b(k_l, f)}{1 + g \sum_{k_l} \sum_j |\tilde{J}_j(f)|^2 \delta\left(k_l - \frac{f}{c} \sin\theta_j\right) * b(k_l, f)}. \quad (31)$$

The maximum array sensitivity is along the tilted locus in spatio-temporal frequency space corresponding to the desired signal AOA, with a bandwidth tuned to the power spectrum of the steering signal $|D(f)|^2$ (actually flattened by the LMS dynamics)—having achieved the full array gain of Nd , as the beamforming operation has a linear phase that time delays the arriving signal to time align it with the known reference signal. Jammer nulling is accomplished by an inverse filter which acts as a power nuller to any jamming signals present in the signal environment at their respective angles of arrival (denoted by the $\delta\left(k_l - \frac{f}{c} \sin\theta_j\right)$ term) and frequency spectrums. The total null depth of the processor is limited by the effective loop gain g , and the available null depth is distributed among the number of jamming signals present.²⁹ The grating within the photorefractive processor will more quickly respond to higher power jamming signals to create a greater null depth, while smaller power jamming signals will not be nulled as deeply, or as quickly (although the final jammers' power will be the same after adaptation). As in all LMS based power nulling processors, when the signal of interest is strong its corresponding output amplitude will be clamped to the signal power of the desired signal since the $\sum_{k_l} |\tilde{S}(k_l, f)|^2$ term in the denominator of the transfer function will no longer be dominated by the sum of the power of the jamming signals.

Beamsteering is driven by the known component $\alpha\tilde{d}(t)$ of the desired signal detected at the array $r(t) = \alpha\tilde{d}(t) + \alpha'\tilde{d}'(t)$, which can either be time multiplexed or code multiplexed with the spectrally overlapping unknown desired signal $\alpha'\tilde{d}'(t)$. The traveling fringes detector output contains both $\tilde{d}(t)$ and $\tilde{d}'(t)$ weighted proportionally to α and α' respectively, since they are broadcast by a single transmitter and arrive on the array with identical phase fronts. At steady-state the amplitude of the known component at the differencing node adaptively adjusts to a nearly exact match of the known reference signal; thus just after the differencing node, the known component is cancelled and the unknown desired signal $\alpha'\tilde{D}'(f)$ remains detected with the full array gain and all of the jammers optimally extinguished.

3.9 BEAMTAP system simulations

We simulated the BEAMTAP algorithm to demonstrate its operation for broadband squint free beamforming and nulling and to verify the spatio-temporal frequency domain transfer function formalism. Figure 8 characterizes the input signals in spatio-temporal Fourier space with transverse spatial frequency along the vertical axis and a single-sided temporal frequency along the horizontal axis. The desired signal is a broadband Gaussian apodized chirp whose $1/e$ spectrum spans the frequency range from 1.1 GHz to 1.9 GHz. Its AOA is set at 0.25 radians and it falls on a tilted locus in spatio-temporal frequency space. Jammer 1 is a broadband filtered Gaussian white noise signal, spanning the frequency range from 0.5 GHz to 2.5 GHz, with an AOA of -0.2 radians. Jammer 2 is a narrowband sine wave, set at a frequency of 0.8 GHz and with an AOA of 0.5 radians. Jammer 2 is selected

such that it lays on the maximum of the first sidelobe of the receptivity pattern of the chirp when beamforming is performed without jammers. In the simulations presented here the power in each jammer is one thousand times stronger than the power in the desired signal. However, the power of all signals was normalized in Fig. 8 for illustrative purposes in order to make all the signals visible.

The diagram shown in Fig. 9 illustrates the various facets of the simulation. The leftmost figure shows the spatio-temporal RF field amplitude detected by the array antennas sampled at 125 ps (i.e. with a frequency $f_s = 8$ GHz) along the horizontal time axis and spatially sampled by the 64 antenna elements along the vertical axis. Notice that only the jammers are visible, since they are much stronger than the signal. The final weight values after adaptation for 300 μ s with a net loop gain of 1 are shown in the center of this figure. Notice also that even though the signal is totally buried by the jammers, the dominant feature which builds up in the weight is the tilted stripe which results from the correlation between the desired component of the feedback signal $f(t) = d(t) - o(t)$ (the difference between the desired chirp signal and the output) and the input signals $s_n(t)$ (chirp and jammers) whose tilt is an indication of the angle-of-arrival of the desired signal. Weak sidelobes seen in the weight matrix are due to a cross-correlation between the signal and jammers and are responsible for the manipulation of the antenna nulls to point towards any undesired jammers. The instantaneous input signal vector from the array is multiplied (eg diffracted vertically) by this weight matrix in order to transform array element positions into topologically ordered spatial positions arranged with linear time delays at the velocity of the traveling fringes detector; detection on the traveling fringes detector thereby compensates for the unknown time delays to each antenna element of the desired signal. The diffracted signal is deflected vertically where it is detected and accumulated in the scrolling detector array, which in this simulation was shifting at a sampled rate of 8 GHz. The upper part of the figure shows the contents of the traveling fringes detector at this instant, showing the accumulations of the light diffracted by the photorefractive weights, which generates the system output $o(t)$ after propagating fully across the detector aperture. The lower part of the figure shows the contents of the scrolling input modulator (AOD) at this instant in time. An outer-product (eg holographic interference) between this signal in the AOD and a delayed version of the input signal, is used in order to adapt the weights. This simulation used an array with 64 antenna elements and 64 taps in the delay line.

Figures 10 and 11 show the receptivity pattern as a function of AOA and temporal frequency of the system for two different cases, first with only the signal present in the environment, and then in the presence of the strong jammers as well. The receptivity patterns were produced by simulating narrow Gaussian pulses incident upon the phased array coming from different AOAs, according to the expression $T(\theta, f) = O(f)/G(f)|_{g_n(\theta, t)}$, where $G(f)$ represents the spectrum of the incident Gaussian pulse, $g_n(\theta, t) = e^{-(t-n\frac{\Delta}{c}\sin\theta)^2/\Delta^2}$ represents the Gaussian pulse detected by each antenna element, and Δ represents the $1/e$ pulse width. Each pulse is propagated through the BEAMTAP system (using frozen weights) to provide the corresponding output. The output is Fourier transformed and is then divided by the spectrum of the initial Gaussian pulse, providing us with the transfer function for a given AOA θ . Repeating this process over several AOAs gives the final transfer function as a

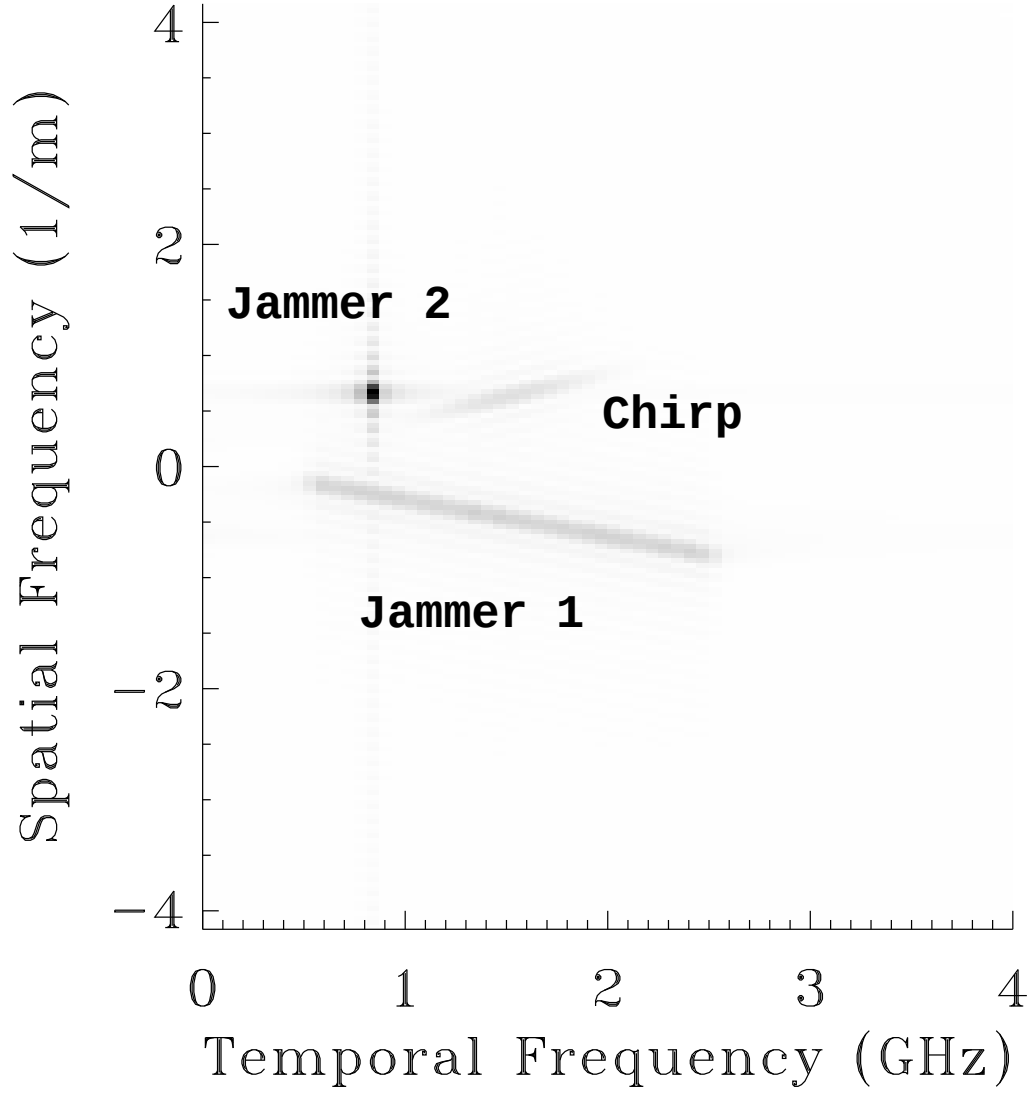


Figure 8: Spatio-temporal Fourier space representation of the input signals used in the computer simulation of the BEAMTAP algorithm which were selected in order to test the system's jammer nulling capacity over a diversified signal environment. The desired signal is a broadband Gaussian chirp. Jammer 1 is a broadband filtered Gaussian noise and jammer 2 is a single frequency sine wave, each of which are 30 dB stronger than the signal of interest.

function of θ and f in terms of the weights

$$T(\theta, f) = \sum_{n=1}^N \sum_{\tau=0}^{M-1} W_{\tau n} e^{-i2\pi f(t_s \tau + n \frac{d}{c} \sin \theta)}, \quad (32)$$

where t_s is the sampling period. This function is a coordinate transformed version of the

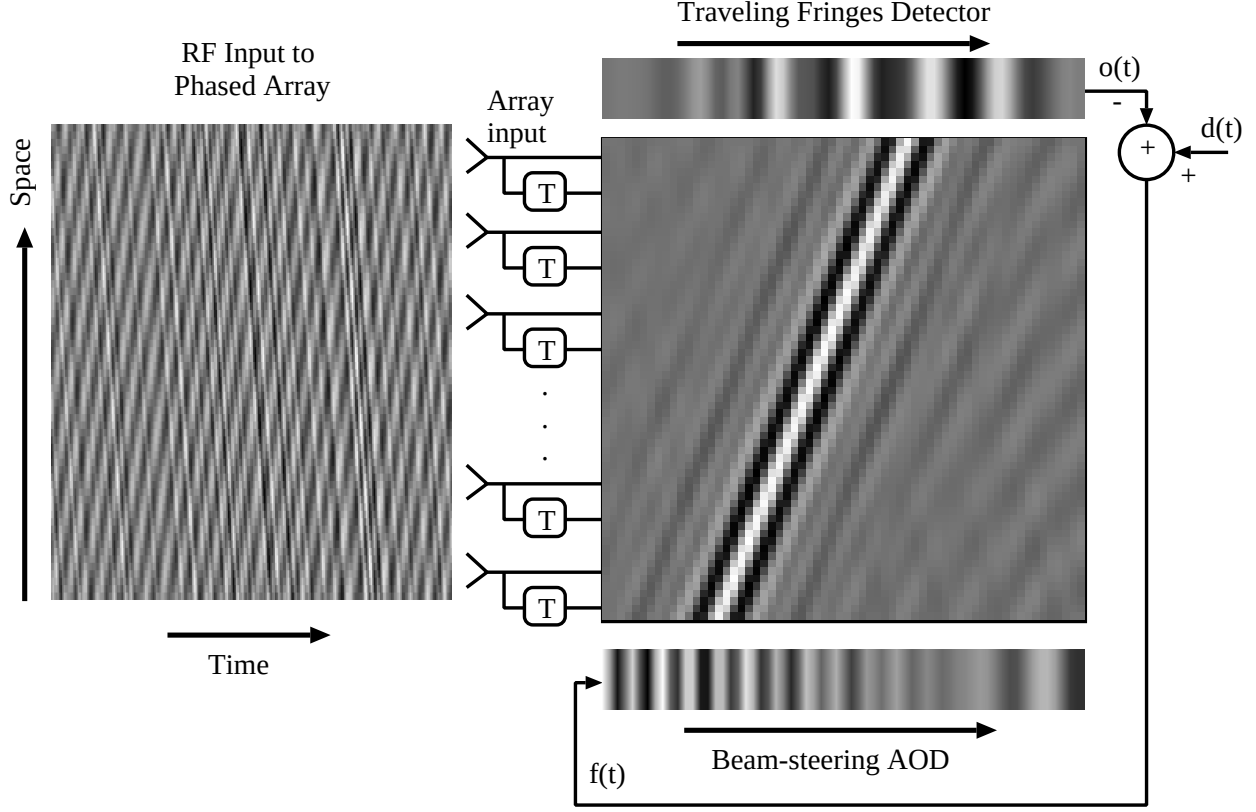


Figure 9: Diagram illustrating the simulation of the BEAMTAP architecture. The input signal is represented by the array of time histories shown at the left. At every time-step an instantaneous slice of the input is detected by the antenna arrays and is propagated through the adaptive weight matrix (center of figure). The product of the input vector with the weight matrix is diffracted vertically and detected and accumulated on the scrolling detector (top of figure). The output $o(t)$ is subtracted from the desired signal $d(t)$, generating the feedback signal $f(t)$, which is fed through the scrolling delay line (bottom of figure). An outer-product between the scrolling feedback signal and a delayed version of the input is used to adapt the weights, producing the resulting tilted cross-correlation slice seen in the weight matrix in the center of the figure at steady-state.

spatio-temporal transfer function derived in Eq. 31, where the tilted loci in the spatio-temporal frequency representation are mapped onto the orthogonal sampling of the AOA-temporal frequency representation.

In Fig. 10 beamforming is performed only with the desired repetitive chirp in the input, without the jammers. Notice the strong response in the direction of the desired signal (0.25 radians), over almost the entire chirp bandwidth (0.5 GHz to 2.5 GHz), spatially blurred by the angular resolvability of the finite rectangular aperture of the array, which of course varies with frequency. It is interesting to notice that even though the chirp has a Gaussian window, the system transfer function is almost flat over the full bandwidth of the signal. This is explained by the fact that the system adapts in order to produce an output that resembles the desired signal as closely as possible. Since both the input and desired signals are Gaussian

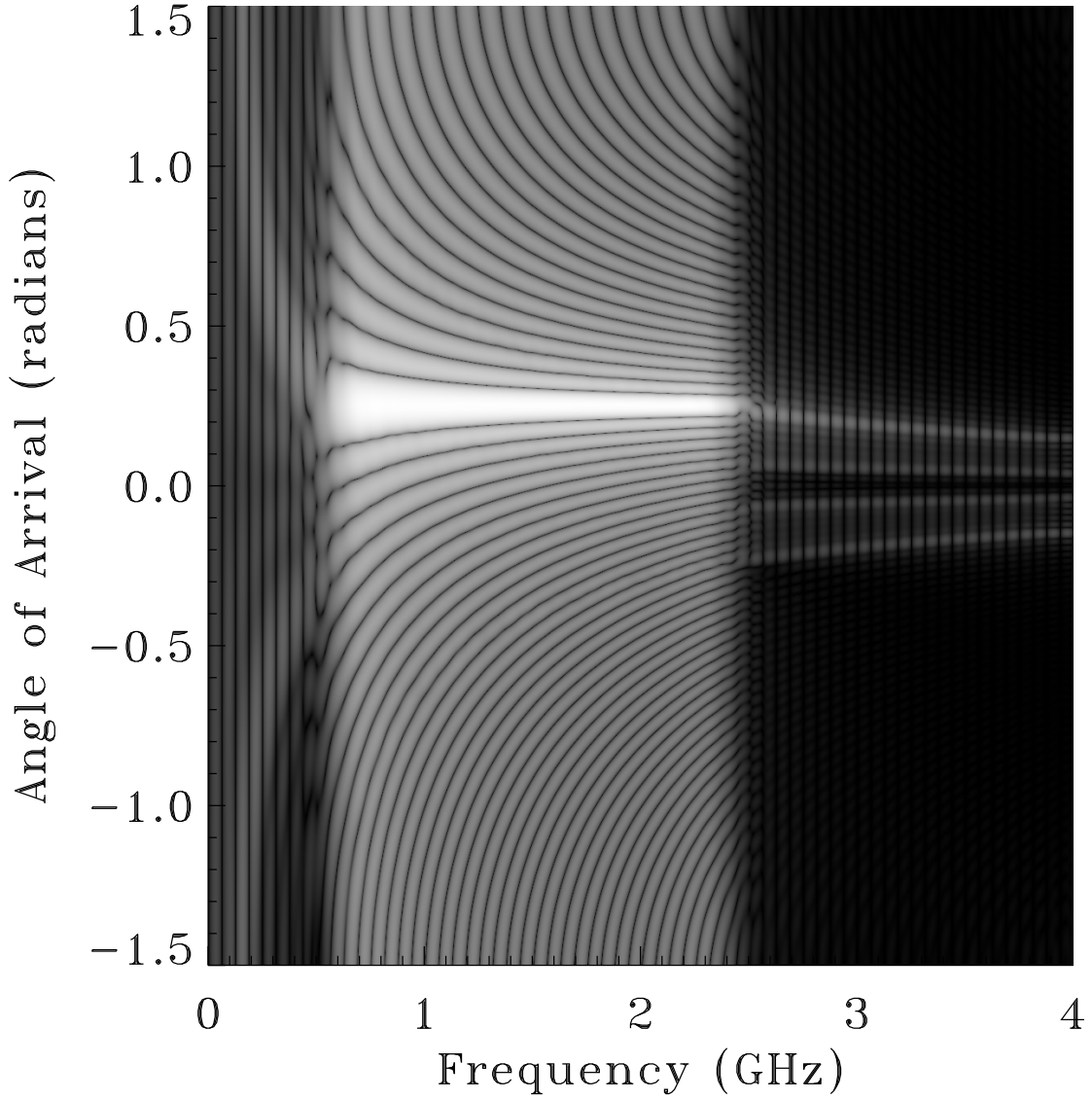


Figure 10: Angle of arrival vs. frequency receptivity pattern that develops after adaptation when only the desired signal is present at the input. Notice that the main lobe does not vary its position with frequency (although its width does change) as it spans the entire input signal bandwidth—thereby demonstrating squint-free true-time-delay beamforming.

chirps, it is to be expected that the system response should be flat over the entire signal bandwidth until noise or jammers start to dominate the signal. Also notice that maximum receptivity describes an untilted line at a constant unsquinted angle which is characteristic of a true-time-delay system. However, the mainlobe width does vary with frequency as do the positions of the sidelobes and nulls. This is to be expected, since adaptation in the presence of signals and antenna noise has optimized the response of the main lobe only.

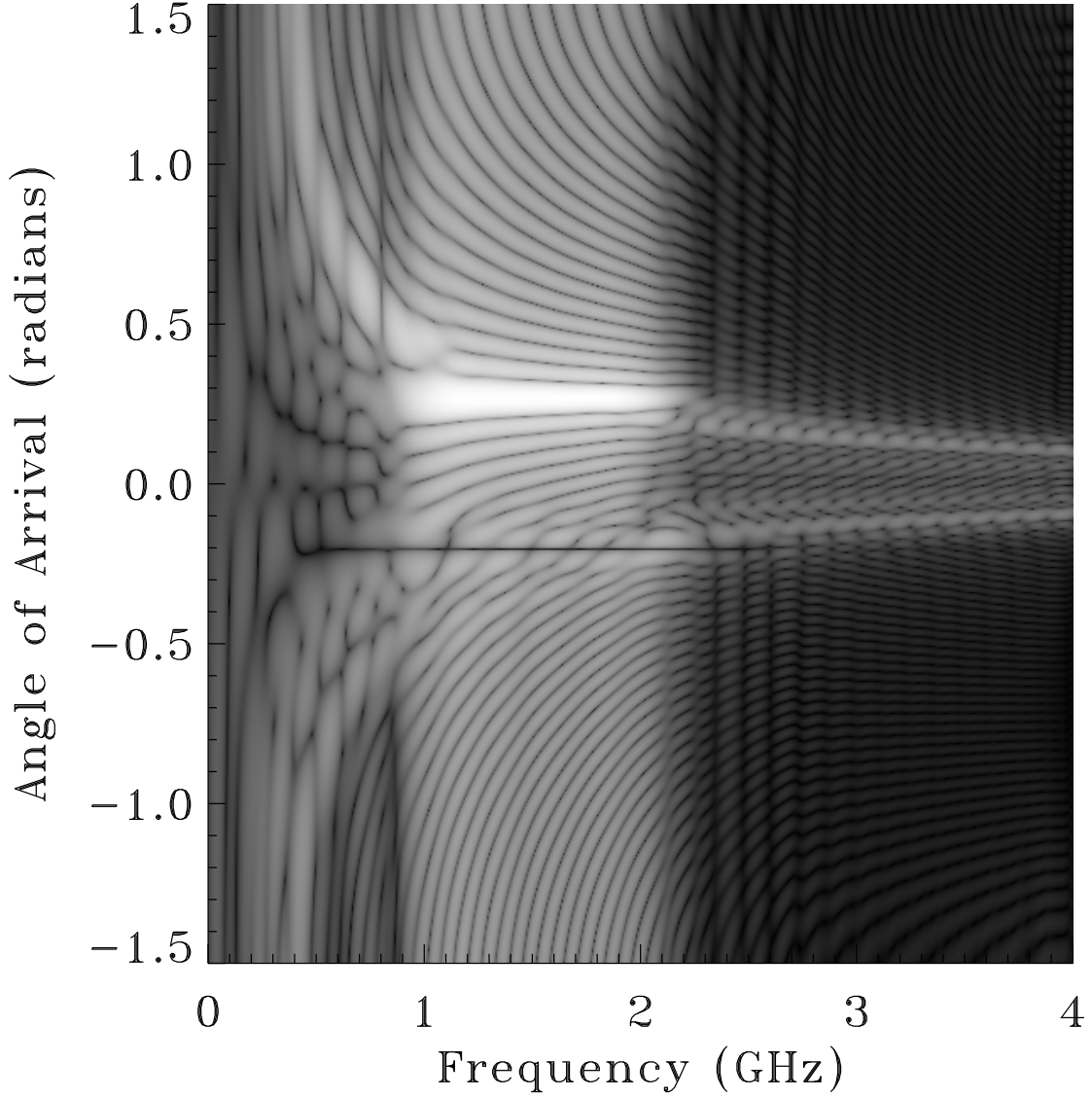


Figure 11: Angle of arrival vs. frequency receptivity pattern after adaptation when the desired signal and strong jammers are both present at the input—demonstrating squint-free jammer suppression with deep nulls. Notice the extremely narrow constant angle squint-free null at the angle of the broadband jammer (-0.2 radians) over its full bandwidth, a narrowband null at 0.5 radians and 800MHz with deep sidelobe nulls, and a slight reduction in the bandwidth of the system response to the desired signal in comparison to Figure 10 (although more than the full $1/e$ bandwidth of the signal is still uniformly detected).

In Fig. 11 both the signals and jammers are present in the signal environment so beamforming and jammer nulling are performed simultaneously. Notice the changes compared to Fig. 10. The main lobe's bandwidth has been slightly reduced, but it still covers the full

$1/e$ spectrum of the signal (1.1 GHz - 1.9 GHz). Part of this reduction is explained by the presence of narrowband jammer 2 at 0.8 GHz, on the first sidelobe of the main beam. Since the spatial sidelobe of this jammer extended onto the main beam and throughout its side-lobes, an improved error performance occurs by producing a null in that frequency, which spans a wide span of AOAs, without any detrimental effect on the signal. The wideband jammer 1 produced a long and thin null, located exactly at its AOA (-0.2 radians), spanning its complete bandwidth (from 0.5 GHz to 2.5 GHz) without any squint of the null, and a width which is significantly less than the main beam of the array. Both jammers were nulled to a depth of 45 dB with respect to the power of the detected signal of interest.

4 Conclusion

We have presented, analyzed, and simulated a new approach called BEAMTAP (Broadband and Efficient Adaptive Method for True-time-delay Array Processing) to broadband beamforming that reduces the number of TDLs required for time-domain adaptive beamforming of a broadband N -element antenna from the conventional value of N TDLs to only 2. For large arrays, this can be a dramatic optical hardware savings that will allow the implementation of broadband adaptive phased arrays in sizes that were previously impractical. The RF photonic implementation of this BEAMTAP algorithm is ideally matched to the strengths of optical phased array processing systems. We have presented an optical architecture based on this new hardware-efficient adaptive algorithm that utilizes a fiber-remoted coherent phased array, a photorefractive crystal, an acousto-optic deflector, and a synchronously scanning traveling fringes detector, whose electronic output represents the adaptively beam-steered and jammer-nulled output. Simulations were presented that verified this operation for broadband desired signals in the presence of narrowband and broadband jammers.

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