

Optical implementation of a single-layer finite impulse response neural network

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ABSTRACT

This paper demonstrates a space integrating optical implementation of a single-layer FIRNN. A scrolling spatial light modulator (SLM) is used for representing the spatio-temporal input plane, while the weights are implemented by the adaptive grating formation in a photorefractive (PR) crystal. Differential heterodyning is used for low-noise bipolar output detection and an active stabilization technique using a lock-in amplifier and a piezo-electric actuator is adopted for long term interferometric stability. Simulations and initial experimental results for adaptive sonar broadband beamforming are presented.

Keywords: optical neural networks, finite impulse response neural networks, adaptive optical processing, differential heterodyne detection, active stabilization, sonar beamforming.

1. INTRODUCTION

Finite impulse response neural networks (FIRNNs) make use of finite impulse response (FIR) adaptive filters as intra-layer interconnects. The added ability of temporal adaptation of a FIRNN elevates its computational complexity from $O(N^2)$ to $O(N^2T)$, where N represents the number of input nodes and T represents the number of time delays in the FIR delay-lines. The cubic order makes the operation of FIRNNs asymptotically intensive, making the parallel optical implementation of these operations attractive.

We have previously proposed various optical architectures for the implementation of FIRNNs.¹⁻³ The first of them uses a time-integrating (TI) optical architecture for the implementation of multi-layer FIRNNs at high frequencies, making use of acousto-optic devices (AOD) for the instantaneous representation of the input nodes and charge-coupled devices (CCD) for the time-integration of the output. However, such an architecture is not capable of on-line learning.¹ Other architectures use both time-integration and space-integration (SI) in the implementation of multi-layer FIRNNs.^{2,3} In these proposed schemes, the TI and SI architectures are folded together to efficiently implement on-line learning, using Wan's temporal back-propagation algorithm.⁴ Zhou and Anderson⁵ described the experimental implementation of a time-delay neural network based on a photorefractive delay-line, making use of associative learning instead of a gradient-descent based algorithm, such as the back-propagation or the least-mean squares (LMS) algorithm.⁶ Therefore, such an architecture is not capable of adaptively compensating for cross-correlations between the desired and unwanted signals.

This paper describes the experimental implementation of a single-layer, linear FIRNN, with the ability of adaptive on-line learning. It makes use of a SI architecture with a PR volume hologram for representation of the adaptive weights, time-multiplexing for read/write isolation, and a differential heterodyne detection technique with active feedback control for bipolar signal detection with interferometric stability. Section 2 gives a brief introduction to the single-layer FIRNN and its learning algorithm, explaining its use in adaptive antenna arrays. Section 3 contains a detailed description of the experimental realization. Section 4 presents computer simulations and the initial experimental results. Section 5 concludes the paper.

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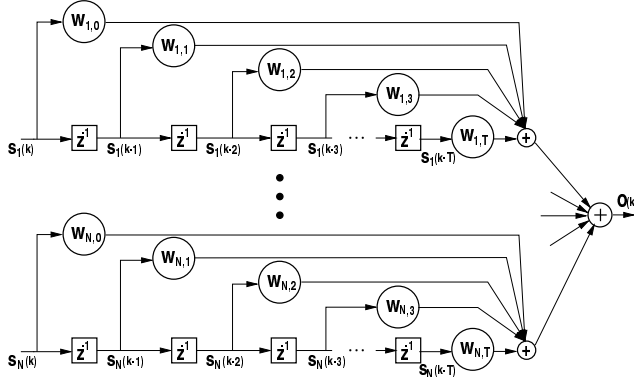


Figure 1. Schematic representation of a linear, single-layer, single-output FIRNN in forward propagation. Squares represent time delays and circles represent adaptive weights.

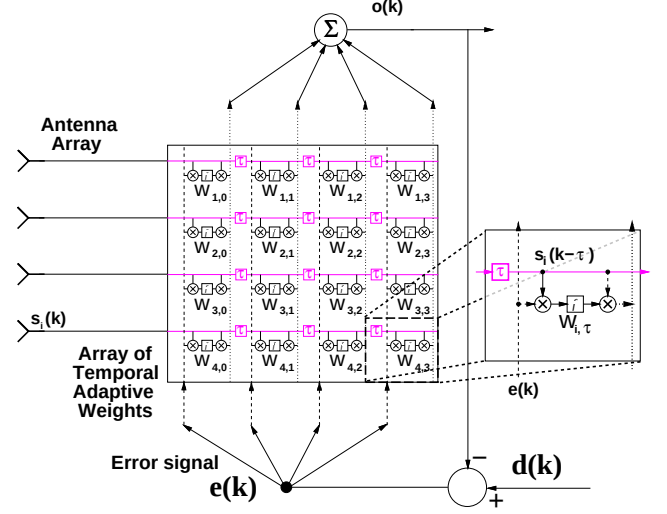


Figure 2. The single-layer FIRNN used in the adaptive processing of signals from an antenna array. The output is subtracted from the desired signal producing the error signal which is used in weight adaptation according to the LMS rule.

2. THE SINGLE-LAYER FIRNN

Figure 1 depicts a linear, single-layer, single-output FIRNN in forward propagation. $s_i(k - \tau)$ represents the input node i at time-step k delayed by τ time steps and $w_{i,\tau}$ represents the adaptive weight interconnecting the input node s_i to the output node y after τ time delays. It is important to note that the present single-layer FIRNN is identical to an adaptive array LMS processor, such as the one described by Widrow.⁷ Note that the interconnections from the input nodes to the output node are comprised of weighted time-delay and sum FIR filters. As such, the output at time step k is given by the following weighted summation of delayed inputs

$$o(k) = \sum_{\tau=0}^T \sum_{i=1}^N w_{i,\tau} s_i(k - \tau). \quad (1)$$

In the linear, single-layer case, the FIRNN back-propagation algorithm simplifies to the LMS rule, *i.e.*

$$w_{i,\tau}(k+1) = w_{i,\tau}(k) + 2\eta e(k) s_i(k - \tau), \quad (2)$$

where $e(k) = d(k) - o(k)$ is the instantaneous error signal, $d(k)$ is the desired signal, and η is the learning rate.

The instantaneous weights can be described by the sequential accumulation of previous weight changes, which can be represented by the correlation between the input $s_i(k)$ and the error signal $e(k)$. That is,

$$w_{i,\tau}(k) = 2\eta \sum_{c=0}^k e(c) s_i(c - \tau), \quad (3)$$

and the output can then be expressed as

$$o(k) = 2\eta \sum_{i=1}^N \sum_{\tau=0}^T s_i(k - \tau) \sum_{c=0}^k [d(c) - y(c)] s_i(c - \tau). \quad (4)$$

This operation describes the correlation cancellation operation of a power nulling spatio-temporal filter,⁸ which finds applications as linear predictors,⁹ and beamformers and jammer nullers for adaptive antenna arrays.⁷ As will

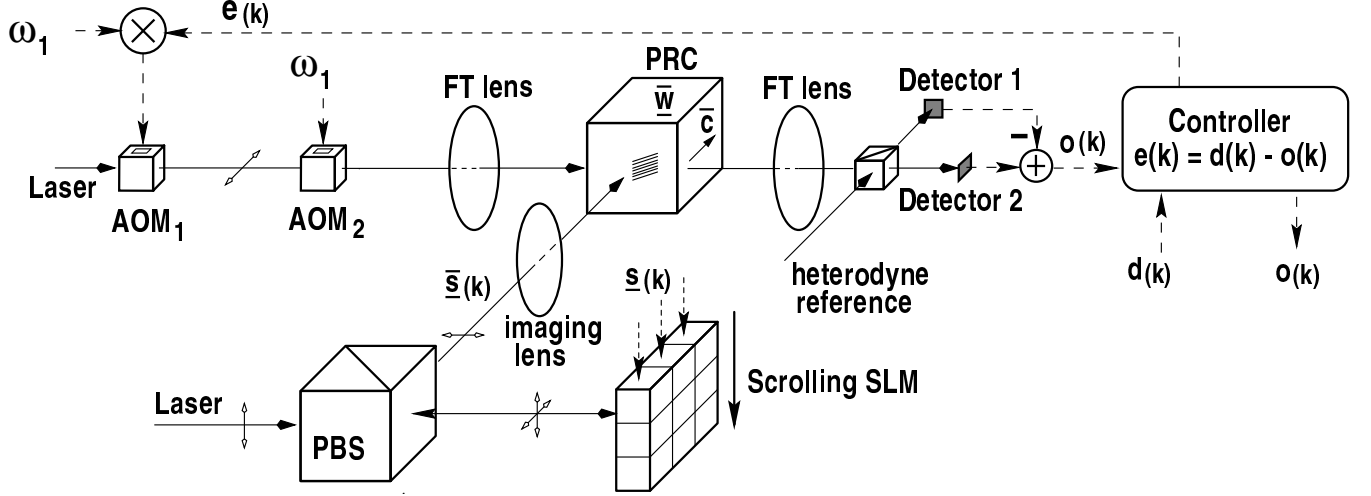


Figure 3. Experimental diagram for a single layer FIRNN using an electronically addressed FLC-SLM in scrolling mode for representation of the spatio-temporal input plane. The adaptive weights are temporally integrated in the photorefractive crystal (PRC). Space integration of the diffracted input produces the final output. The error signal is calculated from the difference between the output and the desired signal. Its product with the input plane in the volume of the PRC modifies the adaptive weights. Solid lines represent optic signals and dashed lines represent electronic signals.

be shown in section 3, our optical architecture performs the same operation in an antenna array, adaptively steering an antenna gain pattern in the direction of the angle-of-arrival (AOA) of a desired signal while simultaneously nulling out potential noise sources and intentional jammers. Figure 2 illustrates the use of a single-layer FIRNN in an adaptive array. The field detected by each antenna element constitute the instantaneous input vector $\underline{s}(k)$. The desired signal $d(k)$ is used to steer the antenna gain pattern to the AOA of the signal of interest and the actual information being communicated is added to $d(k)$.

Optical processors have already been built to address the solution of a similar problem. Hong and Chang presented an implementation scheme using two-beam coupling in a photorefractive (PR) crystal.^{10,11} This technique, however, is limited to narrow band signals. True-time delay is necessary in order to process broadband signals, and the larger the number of time delays, the larger the number of degrees of freedom (DoF) available in temporal processing. Rhodes presented a technique that made use of true-time delay by using acousto-optic devices for time delay and temporal integration of the weights in a liquid crystal light valve (LCLV).¹¹ However, only a single input node was dealt with. The presence of multiple input nodes would require the use of multi-channel Bragg cells, which are limited in practice to about 32 channels. The technique here presented uses the SI architecture described in reference.³ It uses a ferroelectric liquid crystal (FLC) SLM in a scrolling mode (to be described in section 3.1), that limits the highest attainable temporal frequency, but provides a large number of input nodes (256) and a large number of time delays (256), making available a large number of DoF to adaptive processing (*i.e.* 2^{16} DoF for the single output case).

3. EXPERIMENTAL IMPLEMENTATION

The optical architecture used to implement our adaptive system is shown in the diagram in figure 3. It makes use of temporal integration for weight accumulation and spatial integration for output computation. The vector of input data $\underline{s}(k)$ is used to modulate the first row of the scrolling SLM. At every time step the SLM shifts its input by one row, thereby producing a spatio-temporal plane representation of the delayed versions of the input vector. Thus, input element s_i delayed by τ time steps will be represented by the SLM pixel located at position (i, τ) . The mode of operation of this device is explained in more detail in sub-section 3.1. The modulated spatio-temporal input plane $\underline{\bar{s}}(k)$ is diffracted by the dynamic hologram stored in the photorefractive crystal (PRC), representing the interconnection weights. The PR grating accumulation and diffracted output is described in detail in sub-section 3.5. The network output is interferometrically detected and amplified using a differential amplifier, producing the final output $o(k)$.

The differential heterodyne detection and its stabilization are explained in more detail in sub-section 3.4. The signal $o(k)$ is then digitized by a micro-controller, which contains the desired signal $d(k)$ stored in its memory. It calculates the error signal $e(k)$ and feeds it to the RF amplifiers responsible for driving the acousto-optic modulator (AOM). Two AOMs are used, in order to compensate for the Doppler frequency shift of the modulated signal, as explained in sub-section 3.3. The optical beam modulated with the error signal is then used to modify the index grating formed in the PRC, by interfering with the input plane $\underline{s}(k)$, as described in sub-section 3.5. Besides calculating the error signal and communicating its output to a computer, the micro-controller is also responsible for synchronizing the read/write operations using temporal multiplexing, as described in sub-section 3.2. Notice that the operations performed electronically are of unitary order of complexity (the number of operations is not affected by the size of the array or the number of time delays), thus allowing for an efficient scaling of the architecture. More time delays and input nodes can be accommodated simply by using a SLM with a larger number of rows and columns, respectively.

3.1. The scrolling SLM

This section describes in more detail the operation of the scrolling SLM used in figure 3. A Displaytech FLC-SLM with 256x256 pixel elements is used, driven by a SLM driver which can store up to 128 frames and is capable of displaying them in succession at frequencies from 2Hz up to 2.5kHz. The scrolling of the input signal was simulated by first generating the data frames in a computer simulation, then producing the 128 frames representing their sequential scrolling and then transferring the frames to the SLM driver. Thus, a “movie” is generated, which simulates the arrival of a new input data vector at each time step, representing the instantaneous signal detected by the antenna array. The frames are generated such that the 129th frame is identical to the first, thereby enabling the continuous simulation of the arrival of new data without interruption. The SLM is used as a binary phase modulator, which reduces the amount of power in the DC spot. This spot is further spatially filtered in the Fourier plane, since it conveys no information and it would only cause a reduction in the output signal to noise ratio (SNR), by contributing to the decay of the PR grating and increasing the amount of optical power in PR beam fanning.¹²

Figure 4 shows a single frame captured from the SLM in the optical system, representing the delayed versions of an input broadband chirp signal after detection by a 256-element antenna array (the same as the number of pixels in one row of the SLM). Since FLC-SLMs are only capable of representing data in binary format, the resulting quantization of the input data is an unfortunate limitation to our processor, but alternative analog distorted helix or smectic FLC-SLMs could be used to overcome this limitation. Figure 5 shows the Fourier plane of the SLM. Its spectrum is represented by a long line, verifying that the signal is broadband, as expected. The line is thin, pointing at a specific direction, verifying that the input data simulates the detection of a plane wave with its source located in the far-field of the antenna array. Our processor, however, can equally well process signals located in the near field. In this example, the input data consists of a linear chirp with a time-bandwidth product equal to 40, with initial frequency equal to $0.125f_N$ and final frequency equal to $0.75f_N$, where f_N is the Nyquist frequency. The incoming signal AOA is 0.2 radians and the adaptive array has $\lambda_{min}/2$ of spacing between antenna elements, where $\lambda_{min} = f_N/v$, where v is the velocity of propagation of the detected waves in the transmission medium.

The optic field amplitude after SLM modulation is represented by

$$E_{i,\tau}^1(x, y, t) = a_1 \eta_S E_0 e^{i\omega t - kz} \sum_{i=1}^N \sum_{\tau=0}^{N-1} \prod \left(\frac{x - i\Delta_x}{\Delta_x} \right) \prod \left(\frac{y - \tau\Delta_y}{\Delta_y} \right) s_i(t - \tau\Delta_t) \quad (5)$$

where a_1 is the fraction of the initial field amplitude incident on the SLM after beam-splitting, η_S is the diffraction efficiency of the SLM, $\prod(\cdot)$ is the rect function, Δ_x and Δ_y are the pixel pitch in the x and y directions, and Δ_t is the time interval between time steps, determined by the frame rate selected on the SLM driver. We have set it at 1kHz and, therefore, $\Delta_t = 1\text{ms}$. Thus, the current time step is given by $k = t \text{ DIV } \Delta_t$, where DIV denotes integer division.

3.2. Temporal multiplexing

Referring back to figure 3, the input plane is diffracted by the adaptive PR hologram to the output detectors. The diffracted output travels along the same optical path as the beam in the AOM arm. This suggests that a method is required to distinguish the diffracted output from the undesired beam which propagates straight through the AOM arm. Different techniques using polarization and angle multiplexing have been proposed for this goal.¹³ However, since we are already using a micro-controller for calculating the error signal, we decided to also use it to control

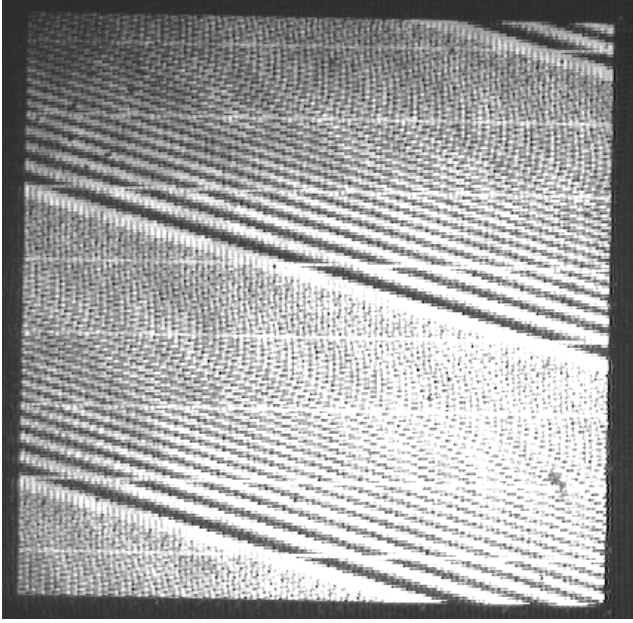


Figure 4. SLM frame captured in the image plane, displaying a continuous broadband binary chirp.



Figure 5. Fourier plane representation of the modulated signal from the SLM, showing that the signal is temporally broadband, repetitive, and that its source is located in the far-field of the antenna array.

the temporal multiplexing of the read and write operations, using the SLM driver trigger output for synchronizing the operation. This is done as follows: time is equally divided between read/write cycles. During the read cycle no signal is sent to the AOM arm, thus shutting off that beam. Consequently, the diffracted signal can be detected by the heterodyne detector without interference. This cycle, denoted t_R , lasts for N time steps. Meanwhile, the error signal is calculated and stored in memory. Next, the write cycle is initiated, during which the output of the detectors is ignored and the calculated error signal is fed to the AOM, modifying the recorded index grating in the PRC, as described in sub-section 3.5. This cycle, denoted t_W , also lasts N time steps. In our case, $N = 256$, $\Delta_t = 1\text{ms}$ and, therefore, $t_R = t_W = 256\text{ms}$.

This scheme results in a delay between the calculation of the error signal and its use in feedback. Such a delay is allowable because after N time steps the input plane is identical to what it was at the time when the error was calculated. As long as the grating exponential erasure is kept small and the signal environment does not change considerably during this time frame, convergence is guaranteed. Both conditions are met in our system. In a real-time processing scenario it would be better to use an alternative read/write multiplexing technique.

3.3. The AO modulator arm

Figure 6 explains in more detail how the AOMs are used in order to modulate the error signal $\epsilon(t)$ and, simultaneously, generate an intermediate frequency (IF). Two AOMs are used: the first AOM is driven by a signal described by $s_1(t) = A_1 \epsilon(t) \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$, where A_1 and A_2 are RF carrier amplitudes, ω_1 and ω_2 are RF carrier angular frequencies. In our case $\omega_1/2\pi$ is set to 100MHz, which is the frequency at which our AOMs have the best diffraction efficiency. $\omega_2/2\pi$ is slightly offset from $\omega_1/2\pi$, at 99.9MHz. Synthesized signal generators were used in order to generate very stable RF carriers ω_1 and ω_2 , respectively. Their stability is important in our application, since the single side band (SSB) beat tone produced by them is used in active stabilization, explained in sub-section 3.4. The field after AOM₁ is given by

$$E_a(x_o, t) \cong E_a \left(t - \frac{x_o}{v_a} \right) = a_2 G \eta_1 g(x_o) E_0 e^{i(\omega t - kz)} \left[\tilde{s}_1 \left(t - \frac{x_o}{v_a} \right) + c.c. \right] \quad (6)$$

where $g(x_o)$ represents the gaussian profile of the illuminating beam, $\tilde{s}_1(t)$ is the SSB analytic signal version of $s_1(t)$, and v_a is the speed of the acoustic waves in the AOMs. In our case, the bandwidth of $\epsilon(t)$ is limited to a few kHz,

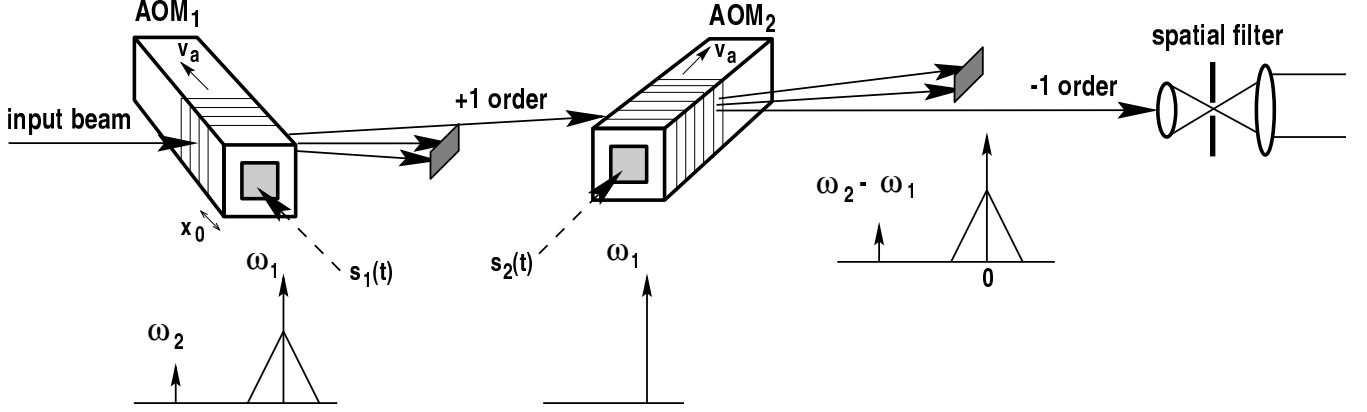


Figure 6. Diagram explaining the use of AOMs to modulate the error signal and produce a SSB tone.

making $s_1(t)$ narrowband in comparison to its carrier frequency. Therefore, we can ignore the angular separation caused by the sidebands of $\epsilon(t)$. $a_1 E_o$ is the incident field amplitude, η_1 is the AOM amplitude diffraction efficiency, and G represents the electronic gain in the feedback. We also assume that the modulating signal power is small enough so that we are operating in the AOMs linear regime.

From the diagram in figure 6, note that only the first positive diffracted order of AOM₁ is allowed to pass, preventing the undiffracted and conjugate term $[\hat{s}_1^*(t)]$ from reaching the second AOM. Thus, $E_a(t)$ can be written as

$$E_a(x_o, t) = a_1 G \eta_1 g(x_o) E_o e^{i(\omega t - kz)} \left[A_1 e \left(t - \frac{x_o}{v_a} \right) \exp[-i\omega_1(t - \frac{x_o}{v_a})] + A_2 \exp[-i\omega_2(t - \frac{x_o}{v_a})] \right]. \quad (7)$$

This field is diffracted by AOM₂, driven by the single RF tone $s_2(t) = \cos(\omega_1 t)$. After AOM₂, only the first negative diffracted order is allowed to pass. Since Δt is large compared to the acoustic propagation delay, we may ignore it, giving us

$$E_2(x_o, t) = a_1 G \eta_1 \eta_2 g(x_o) E_o e^{i(\omega t - kz)} [A_1 \epsilon(t) + A_2 \exp[-i(\omega_2 - \omega_1)t]] . \quad (8)$$

The modulated signal $\epsilon(t)$ is base-band modulated. The tone at ω_2 is mixed with the carrier ω_1 in AOM₂, generating a single side band (SSB) tone at the IF $(\omega_2 - \omega_1)$, which will be used for the active stabilization of the drifts in the interferometric system.

3.4. Differential heterodyne detection and active stabilization in detection

In our setup we observed a strong background noise associated with fluctuations in the intensity of our laser, known as relative intensity noise (RIN). Figure 7 shows a technique which allows us to eliminate noise that, such as RIN, is common to both interferometric arms. Energy conservation at the dielectric interface guarantees that the signals detected by detectors 1 and 2 must be π radians out of phase. Therefore, it is possible to eliminate DC and common mode RIN by subtracting the detector outputs. This operation is performed by a differential amplifier, which produces the output $V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$. The potentiometers allow the amplifier to have a variable gain control, and allow for the balancing of small differences in the DC level of the output due to misalignment and different responsivity of the two detectors. For a signal modulation of $m(t)E_o$, the amplifier output is given by

$$V_{out} \propto |E_o|^2 \{ |m+1|^2 - |m-1|^2 \} = 2|E_o|^2 m(t) + c.c. \quad (9)$$

where the DC and RIN terms have been cancelled.

Heterodyne detection requires a constant phase difference between the reference and the signal beam. Various sources, such as mechanical vibrations, air currents, temperature changes and micro-creep of the optic mounts produce phase drift. Passive forms of phase stabilization that we have used include the use of a pneumatic table to isolate the setup from mechanic vibrations, replacing unstable optic mounts, and covering the optic setup with a thick plastic cabinet. These measures limited our phase drift to about π radians over a time period of half an hour. The effect of the residual phase drift depends on whether the detected signal is SSB or double side band (DSB) modulated. For a DSB signal, the field amplitude of the signal beam is represented by $\frac{E_s}{2} \cos(\omega_s t) e^{i(\omega t + \phi_s)} + c.c.$, where w_s is

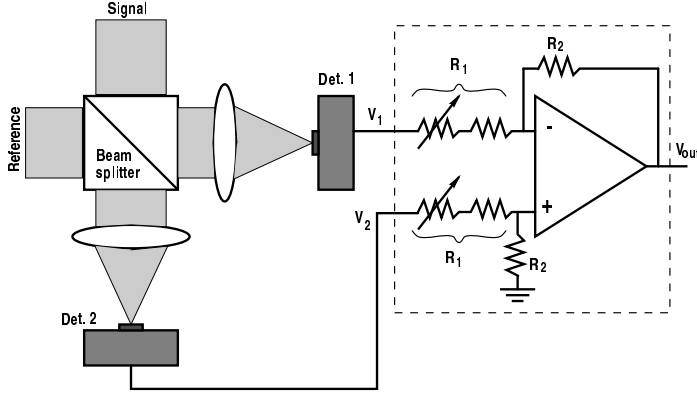


Figure 7. Differential heterodyning. The signals detected by detectors 1 and 2 are subtracted by the differential amplifier, removing common noise sources and increasing the dynamic range of the detected signal.

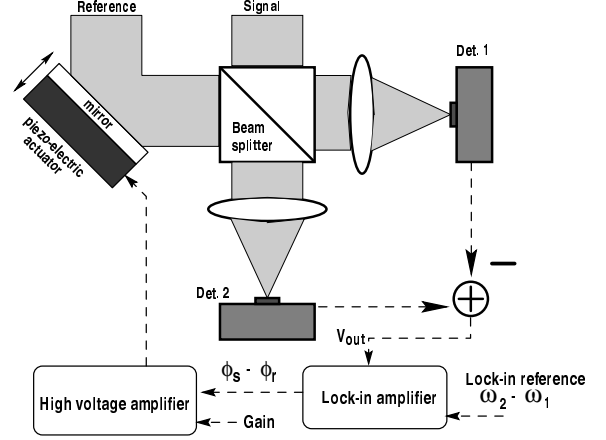


Figure 8. Active stabilization. The lock-in amplifier determines the phase of the reference tone by comparing it to an external lock-in reference.

the angular frequency of the signal of interest and ϕ_s is its initial phase. After interfering with a reference beam $\frac{E_r}{2} e^{i(\omega t + \phi_r)} + c.c.$, the heterodyne signal of interest $E_s(t)E_r(t)^* + c.c.$ becomes $E_s E_r \cos(\omega_s t) \cos(\phi_s - \phi_r)$, that is, the signal amplitude is modulated by the cosine of the phase difference $\phi_s - \phi_r$ and, therefore, this difference must be kept close to zero. A SSB signal, on the other hand, is represented by $\frac{E_s}{2} e^{i\omega_s t} e^{i(\omega t + \phi_s)} + c.c.$ and the heterodyne signal of interest becomes $\frac{E_s E_r}{2} \cos(\omega_s t + \phi_s - \phi_r)$. That is, the phase difference causes a phase shift of the detected signal, but it does not affect its amplitude.

As explained in sub-section 3.3 the AOM arm of the optical processor is DSB modulated with our signal of interest plus a 100kHz SSB tone. Initially, the system is aligned in order to give us maximum amplitude at the DSB modulated signal, at which point the phase difference $\phi_s - \phi_r$ equals to zero. Passive stabilization guarantees us that this signal will be stable for a few seconds and provides us with enough time to initiate active stabilization. Active heterodyne stabilization is performed by a Stanford Research Systems SR850 DSP Lock-in amplifier in combination with a mirror attached to a piezo-electric actuator, as shown in figure 8. The RF carriers are mixed and the resulting 100kHz tone is used as a reference by the lock-in amplifier, which detects the phase of the SSB detected signal. Because of the high gain associated with the lock-in, this tone can be set at 40dB below the DSB modulated signal. The low amplitude of the SSB tone and its lack of correlation with the SLM signals prevents it from interfering with the processor adaptation. The time constant of the lock-in amplifier is set at 30ms, which is fast enough to compensate for most of the residual phase drift, and which is slow enough to provide the lock-in with the gain and filtering necessary to reliably detect the 100kHz tone. The lock-in's phase output, proportional to $\phi_s - \phi_r$ is then connected to a high-voltage amplifier through a voltage divider, which gives us control of the gain. This amplifier drives the piezo-electric crystal, moving the mirror back and forth in the reference arm of the heterodyne detector, correcting for any drifts in the phase. Note that such a system stabilizes only the heterodyne detection process and does not affect the holographic grating formation in the PR crystal. However, for the slow phase drifts we have in our system, the crystal is capable of adaptively forming new gratings whenever the SLM arm and the AOM arm experience a phase change. The heterodyne detector, on the other hand, would cease to work whenever its phase difference reached π radians, and it would remain in that state for a time period over several minutes. That is the reason why active stabilization is so important in heterodyne detection, although it is not mandatory for PR grating formation.

3.5. Photorefractive adaptive grating formation

During adaptation, the coherent superposition of modulated optical signals from the SLM (the spatio-temporal input plane $\underline{\hat{x}}$) and the AOM arm (the error signal $\epsilon(k)$) modify the index grating in the PR crystal. A ferroelectric Ce-doped SBN:75 crystal with significantly large time-constant (of the order of a few seconds at CW illumination of about 100 mW/cm²) has been chosen. In the absence of an external field, and in the limit, with the grating spacing being large compared to the carrier diffusion length, the time response of these crystals typically exhibit inverse dependence on the dielectric relaxation constant¹⁴ and it is, therefore, slow. However, the adaptation time of the

processor can be fast independently of the PR time-constant. This is possible thanks to high electronic gain, which provides us with a high detected signal power even when the grating is very weak, being therefore unnecessary to wait until the grating reaches its steady state.

A near optimal recording geometry is established by maximizing the steady-state diffraction efficiency for an average writing beam intensity ratio equal to ten and neglecting the intensity-dependent effect of the gain coefficient (Γ) associated with the PR crystal. Using transient analysis for grating formation and ignoring some of the higher-order complex PR phenomena, we formulate the PR recording dynamics during the adaptation in the processor in terms of an equation representing the index-modulation as

$$\Delta n(x, y, z, t) = \beta \int_{-\infty}^t E_1(x, y, z, t') E_2^*(x, y, z, t') e^{-\frac{(t-t')}{\tau_{pr}}} dt' + c.c., \quad (10)$$

where β is the crystal sensitivity (the change in index of refraction per unit energy absorbed per unit area), assumed to be constant, E_1 , and E_2 are the amplitude distributions of the writing beams from the SLM and AO arms respectively, and τ_{pr} is the time response of the PR crystal, also assumed to be constant. The integral in the above equation, represents superposition of all the gratings that are scaled by an exponential decay factor $\exp(-\frac{(t-t')}{\tau_{pr}})$ reflecting the finite time response of the PR grating. The index-grating representing the volume hologram in equation 10 consists of a series of gratings produced by the interference of plane wave components of the writing beams. The coupling or diffraction efficiency associated with the incident plane-wave components $\vec{k}_{i,\tau}$ (in Fourier decomposition of amplitude distribution of the read-out beam from the SLM), can be given using the Born approximation in the weak-diffraction limit, *i.e.*

$$G_{i,\tau}(t) = \int \Delta n(\vec{r}, t) e^{i(\vec{k}_o - \vec{k}_{i,\tau}) \cdot \vec{r}} d^3r, \quad (11)$$

where $\vec{k}_o$ is the wavevector associated with the reference beam from the AO arm. Equation 11 ensures the Bragg matching condition for the incident and diffracted plane wave components pairs. Bragg degeneracy is associated with the inverse volume diffraction problem. Since the degenerate diffracted plane wave components produce moving gratings and zero signal in heterodyne detection using SI, they are not considered in the analysis. As discussed in sub-section 3.2, a read-write multiplexing scheme is used during adaptation. Adaptation of the index-grating can be described in terms of evolution of the grating diffraction efficiency as

$$G_{i,\tau}[(2Np+n)\Delta_t] = C \cdot e^{-\frac{n\Delta_t}{\tau_{pr}}} \cdot \sum_{q=0}^{p-1} e^{-\frac{(p-q)(t_W+t_R)}{\tau_{pr}}} \left[\int_{(q-1)t_W}^{qt_W} e(t') s_i^*(t' - \tau\Delta_t) e^{\frac{t'}{\tau_{pr}}} dt' \right], \quad n = 0, 1, \dots, (N-1) \quad (12)$$

where $C = a_1 a_2 G \eta_1 \eta_2 \eta_S |E_o|^2 \beta$, n is the number of time-steps elapsed since the beginning of the present read cycle, $p \in [1, +\infty)$ is an integer index representing the current read cycle. t_W and t_R are the time-apertures during the write and read cycles, respectively. The term $\exp(-\frac{t_R+t_W}{\tau_{pr}})$ describe the exponential decay of the index-grating during the read and write cycles, and may be beneficial to learning, functioning as a regularizer.¹⁵ The resulting amplitude output of the diffracted signal during the time-multiplexed read-out is given by

$$\begin{aligned} y[(2Np+n)\Delta_t] &= C e^{-\frac{\alpha L}{\cos\theta}} \sum_{i=1}^N \sum_{\tau=0}^T s_i(t - \tau\Delta_t) G_{i,\tau}[(2N(p-1)+n)\Delta_t] \\ &= C' e^{-\frac{n\Delta_t}{\tau_{pr}}} \sum_{i=1}^N \sum_{\tau=0}^T s_i(t - \tau\Delta_t) \sum_{q=0}^{p-1} e^{-\frac{(p-q)(t_W+t_R)}{\tau_{pr}}} \left[\int_{(q-1)t_W}^{qt_W} [d(t') - y(t')] s_i^*(t' - \tau\Delta_t) e^{\frac{t'}{\tau_{pr}}} dt' \right], \end{aligned} \quad (13)$$

where $C' = C e^{-\frac{\alpha L}{\cos\theta}}$, L is the grating interaction length, and α is the absorption coefficient of the PR crystal. Equation 13 can assume a similar form to algorithmic equation 4 representing correlation-cancellation operation in a FIRNN, by representing it in discrete form, *i.e.*

$$y^p(k) = C' \gamma_1^{k-2Np} \sum_{i=1}^N \sum_{\tau=0}^T s_i(k - \tau) \sum_{q=0}^{p-1} \gamma_2^{(p-q)} \int_{(q-1)t_W}^{qt_W} [d(t') - y(t')] x_i^*(t' - \tau\Delta_t) e^{\frac{t'}{\tau_{pr}}} dt', \quad (14)$$

where $\gamma_1 = e^{-\frac{\Delta t}{\tau_{pr}}}$ represents the PR grating decay during a single time-step, and $\gamma_2 = e^{-\frac{t_W + t_R}{\tau_{pr}}}$ represents the grating decay in a read/write cycle. Note that p , the current read cycle, can be expressed as $p = 2N \bmod k$ and, therefore, equation 14 is a function of the current time step only. The term γ_1^{k-2Np} indicates that the learning rate will be slightly higher at the beginning of each read cycle. To minimize such an effect, γ_1 should be close to 1, that is, the PR time constant τ_{pr} should be much larger than the time step Δt , a condition which is readily met in our system. The term $\gamma_2^{(p-q)}$ inside the inner summation reflects the exponential decay of previously written gratings, indicating that additional gain is necessary in order to drive the weights to their optimum value. This effect, however, can be advantageous in the presence of dynamically varying input signals, such as beamforming in the presence of moving sources or time-varying jammers.

4. SIMULATIONS AND EXPERIMENTAL RESULTS

4.1. Simulations

In order to determine the expected system performance, we have simulated the LMS learning algorithm in the presence of many of the same parameters and limitations faced by our experimental implementation. The adaptation of an antenna array with 256 antenna elements and 256 time delays is simulated. The input data is a binary chirp plus white noise 15dB below the input signal power. The desired signal is a binary chirp, but the error signal is an analog waveform, just as in our implementation. Also, the input data is repeated at every 128 samples, simulating the behavior of our scrolling SLM. The desired signal is also repeated after 128 samples, providing us with 2 chirps in our time window, assuring us that there will always be an overlap between the chirps in the input plane and the desired signal. This short delay between chirps usually is not the case in real-life scenarios, but it simplifies the experimental implementation.

The top plot in figure 9 shows the learning curve, describing the mean squared error (MSE) progression at every iteration. The variance of the squared error (VSE) is plotted in a dashed line. This plot verifies that the binary quantization of the input and desired signals does not prevent the system from learning, but white noise limits the lowest achievable MSE. The bottom image shows the final weights after adaptation. Note the two tilted lines. These are the result of the correlation between the input chirp and the error signal, described by equation 3. Two lines are present indicating the presence of two chirps in the time window of the processor, as expected. Their tilt is proportional to the sine of the AOA of the input signal and their position determines the exact number of time delays required in order to compensate for the propagation delay of the detected wavefront from antenna element to antenna element.

The top image in figure 10 depicts the final responsivity pattern of the antenna array, which describes the array's frequency response at different AOAs. Note a strong response at 0.2 radians, the AOA of the desired signal in the input plane. Also note that the response occupies a horizontal line, over a high fractional bandwidth, the result of true-time delay beamforming. At higher frequencies, where the array is undersampled, the antenna gratings are also visible, symmetrically placed around the main lobe, curving towards higher angles at lower frequencies. The comb in frequency is the result of the repetition of the chirp in the time window of the processor, which causes its frequency response to be sampled. Such an effect could be avoided in a real system by placing the chirps farther apart. The solid line in the bottom plot shows the response at 0.2 radians, overlapped by the frequency spectrum of the desired signal (dashed line). Note that the binary quantization spreads out the frequency spectrum of the desired signal, while the repetitive chirp produces its periodic modulation.

4.2. Experimental results

Figure 11 shows two signals detected by differential heterodyning. The top one is the desired signal, a binary chirp identical to the ones used in the SLM moving frames. It is propagated directly through the AOMs to the output, proving the satisfactory differential heterodyning operation. As done in the simulations, two identical binary chirps are placed within the time aperture ($t_W=256\text{ms}$) of the write cycle. As a consequence, we are assured that at least one slice corresponding to the maximum of the correlation grating stays completely within the interaction length in the PR crystal.

The plot shown in the bottom of figure 11 shows the output signal from the processor after adaptation for about 10 write cycles. The output is found to contain the two characteristic binary chirp signals. However, one observes the detected signal to be noisy and to contain amplitude variations localized at different chirp frequencies. The noise

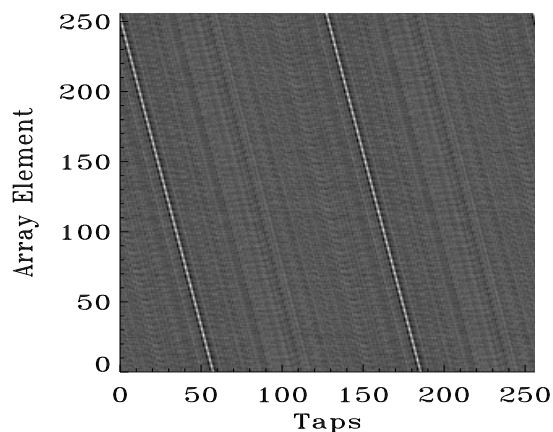
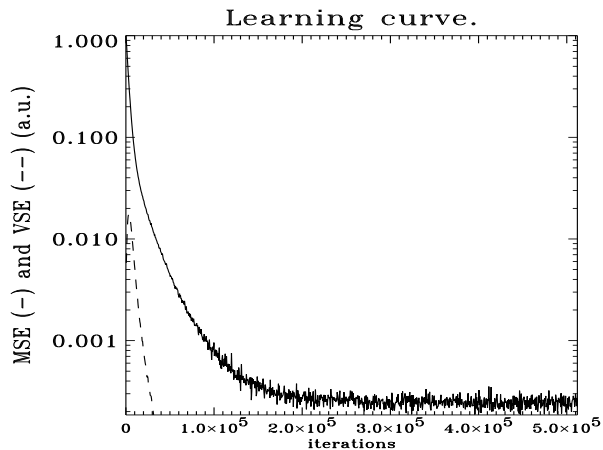


Figure 9. Top – MSE (solid) and VSE (dashed) progression during adaptive learning using the LMS algorithm. Bottom – Final weights showing the correlation between input signal and desired signal.

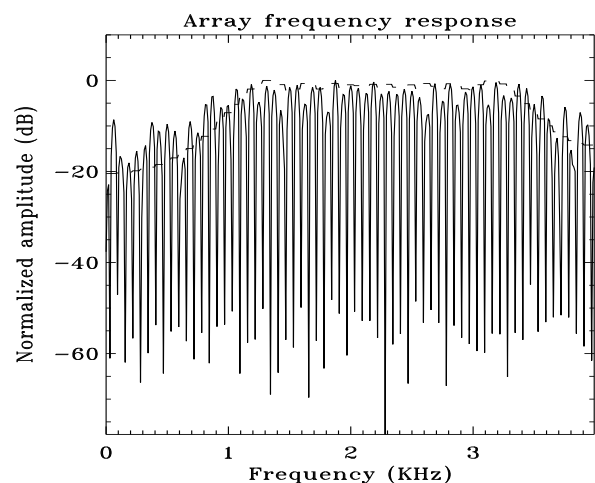
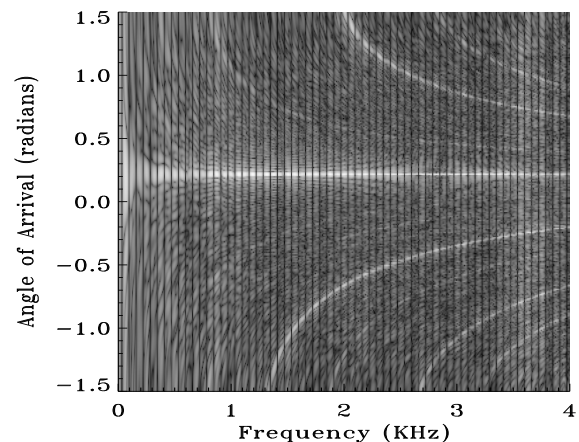


Figure 10. Top – Receptivity pattern after beamforming in the presence of a binary chirp. Bottom – Receptivity pattern in the AOA of the chirp (solid) and binary chirp spectrum (dashed).

is a consequence of thermal noise in the amplifiers and beam fanning. The plot in figure 12 shows the intensity of the cross-correlation between the detected output and the chirp used as the desired signal. The strong central peak, which quickly decays, indicates a strong correlation between the two broadband signals. The two smaller side lobes are a consequence of the chirp repetition. Such a result indicates that broadband beamforming has been satisfactorily achieved, but further work could be applied towards improving the output SNR.

5. CONCLUSION

We have presented the theoretical analysis and demonstrated the optical implementation of a single-layer linear FIRNN applied to the problem of adaptive beamforming in a sonar array. The system makes novel use of a FLC-SLM, making it behave as a scrolling SLM in order to simulate the delayed versions of the input signals detected by an antenna array. At the moment, processing speed in our system is limited by the speed at which the SLM driver downloads a new frame to the SLM. A faster device could be designed for this application, simply requiring the use of a shift register at each modulator column.

We also presented techniques for differential heterodyne detection and active stabilization, making use of a SSB modulated tone in order to detect a DSB modulated signal. Experimental results from the processor were presented for simple beamforming. Further work is currently under progress to demonstrate adaptive jammer nulling.

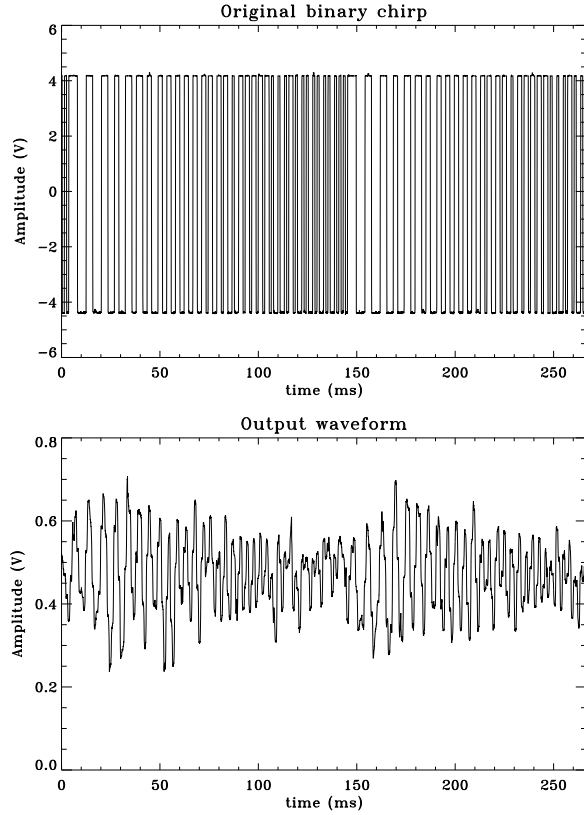


Figure 11. Top – binary chirp (desired signal) detected through differential heterodyne detection. Bottom – processor output, showing the detection of the desired signal.

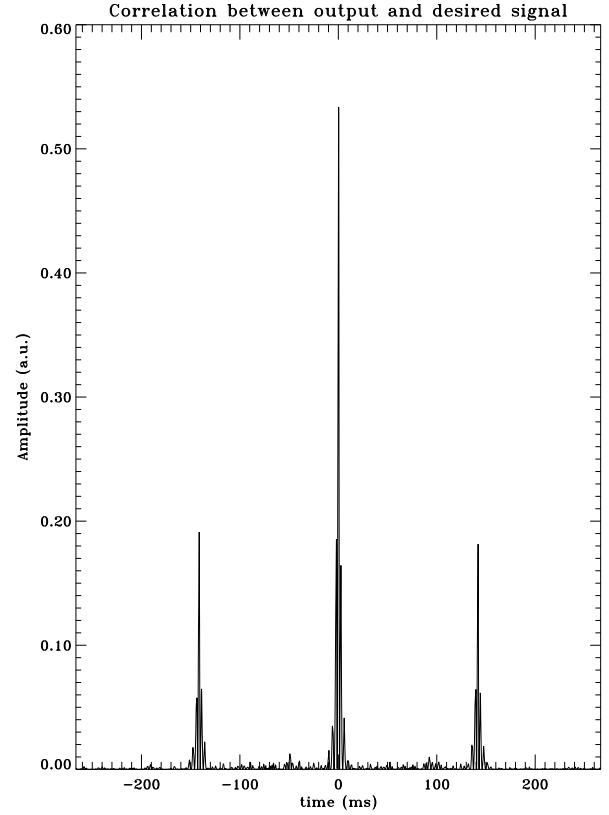


Figure 12. Top – zoom into first 80ms of the output waveform. Bottom – cross-correlation between output and desired signal. Small side lobes are present as two chirps are present during each read/write cycle.

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